CHAPTER

3

Powers and Exponents

Thrill-seekers around the world enjoy the rush of extreme sports like whitewater rafting and kayaking, kitesurfing, hang gliding, rock climbing, heli-skiing, bungee jumping, and sky diving. In some of these sports, the experience involves a free fall. During each second of a free fall, the distance a person falls increases. A formula that approximates the distance fallen is $d = 4.9t^2$, where d is the total distance, in metres, and t is the time, in seconds. In the formula, 2 is an exponent. What does this exponent represent? In what other formulas have you seen exponents?

In this chapter, you will explore the use of exponents in mathematical expressions.

What You Will Learn

- to use powers to represent repeated multiplication
- to solve problems involving powers

Key Words

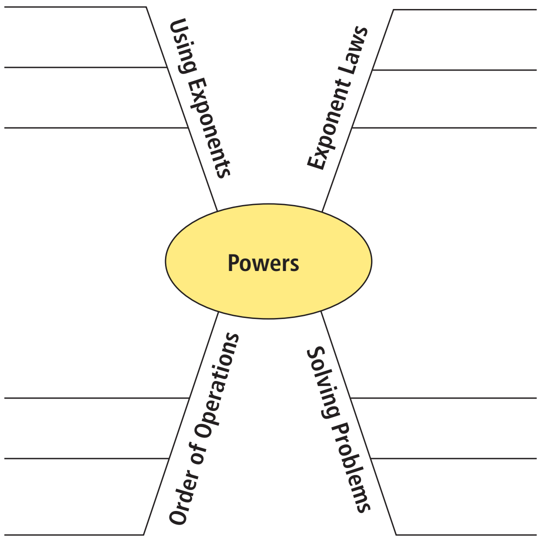
power
exponent
coefficient
base
exponential form

Literacy Link

A spider map can help you understand and connect new terms and concepts. It is designed to be used throughout the chapter.

Create a spider map in your math journal or notebook. As you work through the chapter, complete the map.

- After completing section 3.1, use the upper left leg to identify the parts of a power and the different forms in which powers can be expressed.
- After completing section 3.2, use the upper right leg to list and provide examples of the exponent laws.
- After completing section 3.3, use the lower left leg to list all rules and examples associated with the order of operations involving powers.
- After completing section 3.4, use the lower right leg to list rules and examples related to solving problems involving powers.



FOLDABLES™
Study Tool

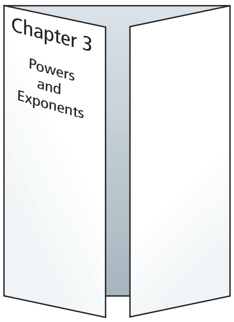
Making the Foldable

Materials

- sheet of 11 × 17 paper
- six sheets of 8.5 × 11 paper
- scissors
- ruler
- stapler

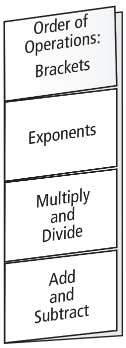
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Label it as shown.



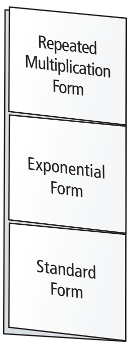
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. Fold in four the opposite way. Make three cuts as shown through one thickness of paper, forming a four-tab book. Label the tabs as shown.



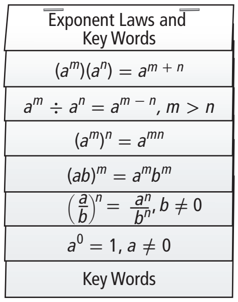
Step 3

Fold the short side of a sheet of 8.5 × 11 paper in half. Fold in three the opposite way. Make two cuts as shown through one thickness of paper, forming a three-tab book. Label the tabs as shown.



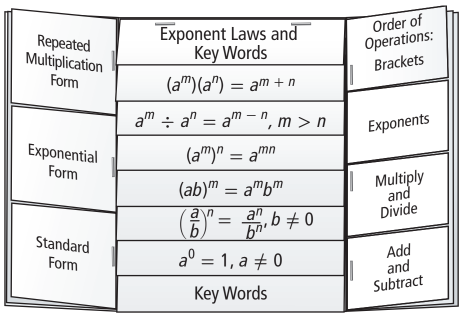
Step 4

Stack four sheets of 8.5 × 11 paper so that the bottom edges are 2.5 cm apart. Fold the top edge of the sheets and align the edges so that all tabs are the same size. Staple along the fold. Label as shown.



Step 5

Staple the three booklets you made into the Foldable from Step 1 as shown.



Using the Foldable

As you work through the chapter, write the Key Words beneath the tab in the centre panel, and provide definitions and examples. Beneath the remaining tabs in the centre, and the tabs in the left and right panels, provide examples, show work, and record key concepts.

On the front of the right flap of the Foldable, record ideas for the Math Link: Wrap It Up! On the back of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

NEL

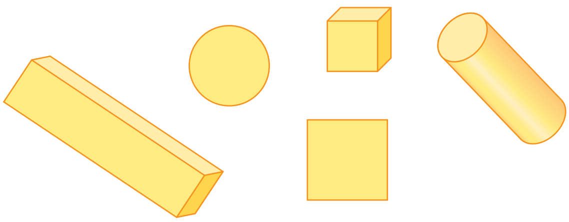
Math Link

Mobile Design

In 1931, Alexander Calder, a mechanical engineer and artist, created intricate sculptures that moved with the air currents of a room. These sculptures are called *mobiles*. Mobiles are known for their bright colours, variety of shapes, and interesting movements. The shapes used are often geometric shapes. Think about some mobiles you have seen. What types of geometric shapes did they contain?

One important consideration when creating a mobile is balance. The balance of a mobile is affected by the surface area and volume of the shapes used. Work with a partner to answer the following questions.

1. What is the name of each geometric shape shown?



2. How could you determine the area of a square?

3. How could you determine the area and the circumference of a circle?

4. How could you determine the surface area of each three-dimensional shape?

5. How could you determine the volume of each three-dimensional shape?

6. How are the methods you suggested in #2 to #5 similar? How are they different?

7. Which shapes would you use to put on a mobile? Why did you choose these shapes?

In this chapter, you will determine the links between geometric shapes and powers by exploring mobile designs. At the end of this chapter, you will design and build a mobile using shapes of your choice.

WWW Web Link

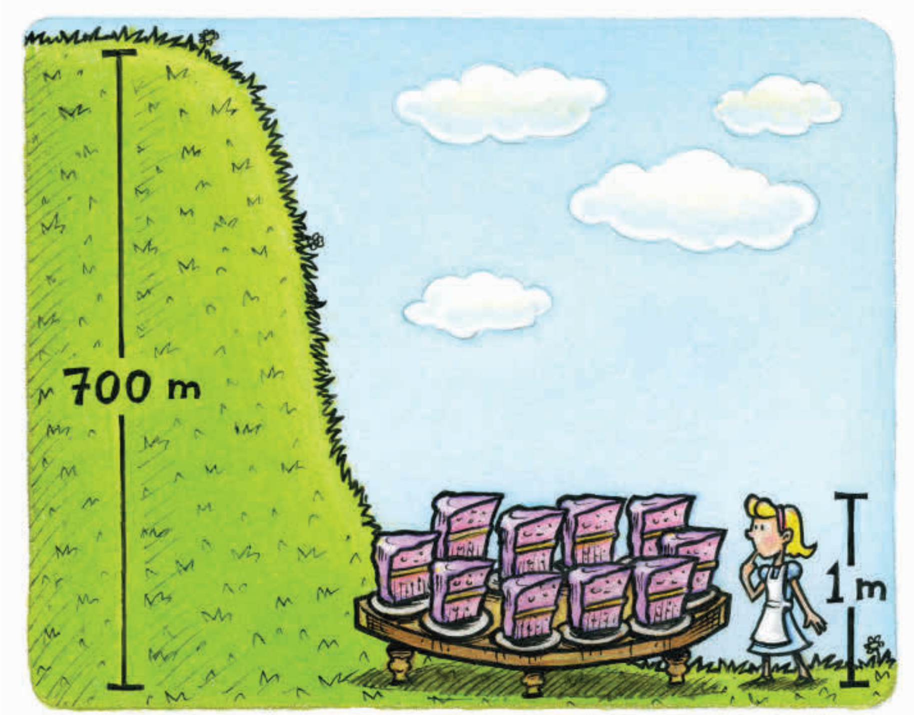
To learn more about mobiles and how to create them, go to www.mathlinks9.ca and follow the links.

3.1 Using Exponents to Describe Numbers

Focus on...

After this lesson, you will be able to...

- **represent repeated multiplication with exponents**
- **describe how powers represent repeated multiplication**



In the story *Alice in Wonderland*, Alice could change her size dramatically by eating cake. If she needed to triple her height, she would eat a piece of cake. Imagine that she is currently 1 m tall. She needs to increase her height to 700 m in order to see over a hill. How many pieces of cake do you think she will need to eat?

Materials

- calculator

Explore Repeated Multiplication

1. Create a table that shows how Alice's height changes after eating one, two, and three pieces of cake. Describe any patterns you see in the table.

2. a) How many pieces of cake does Alice need to eat to become at least 700 m tall? Show how you arrived at your answer.

   b) What is Alice's height after eating the number of pieces of cake in part a)?

   c) How many factors of 3 do you need to multiply to obtain your answer to part b)?

3. Explore how you could use a calculator to determine Alice's height after eating eight pieces of cake. Share your method(s) with your classmates. Record the methods that work for your calculator.

Reflect and Check

4. a) The expression 3^2 can be used to represent Alice's height after eating two pieces of cake. What does this expression mean in terms of factors of 3?

   b) How could you represent $3 \times 3 \times 3 \times 3 \times 3$ as a **power**? Identify the **base** and **exponent**.

5. What is Alice's height after eating ten pieces of cake?

Link the Ideas

Example 1: Write and Evaluate Powers

a) Write $2 \times 2 \times 2 \times 2 \times 2$ in **exponential form**.
b) Evaluate the power.

Solution

a) There are five factors of 2 in the expression $2 \times 2 \times 2 \times 2 \times 2$.
$2 \times 2 \times 2 \times 2 \times 2$ can be written as the power 2^5.
The base of the power is 2 and the exponent of the power is 5.

b) The product $2 \times 2 \times 2 \times 2 \times 2$ is 32.
So, $2^5 = 32$.

Show You Know

a) Write $4 \times 4 \times 4$ as a power.
b) Evaluate the power.

History Link

Euclid was a Greek mathematician who lived from about 325 BCE to about 265 BCE. He was the first person to use the term *power*. He referred to power only in relation to squares.

The first time that the term *power* was used to refer to expressions with exponents greater than 2 was in 1696 in *Arithmetic* by Samuel Jeake.

Euclid

power

- an expression made up of a base and an exponent

base exponent

4^2

power

base

- the number you multiply by itself in a power

exponent

- the number of times you multiply the base in a power

exponential form

- a shorter way of writing repeated multiplication, using a base and an exponent
- $5 \times 5 \times 5$ in exponential form is 5^3

Literacy Link

There are several ways to read powers.

You can read 2^5 in the following ways:

- two to the fifth
- two to the exponent five

Example 2: Powers With Positive Bases

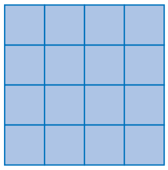
Evaluate each power.

a) 4^2 **b)** 2^3 **c)** 3^6

Solution

Strategies
Model It

a) The power 4^2 can be read as "four squared."
You can use a model of a square to represent the power.



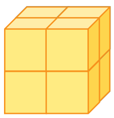
How does the model of the square represent 4^2?

Each side of the square is 4 units in length.
The area of the square is 16 because there are 16 small squares altogether in the square.

In the power 4^2, the base is 4 and the exponent is 2.

$$\begin{aligned} 4^2 &= 4 \times 4 \\ &= 16 \end{aligned}$$

b) The power 2^3 can be read as "two cubed."
You can use a model of a cube to represent the power.



How does the model of the cube represent 2^3?

Each edge of the large cube is 2 units in length.
The volume of the large cube is 8 because there are 8 small cubes altogether in the large cube.

In the power 2^3, the base is 2 and the exponent is 3.

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

c) In the power 3^6, the base is 3 and the exponent is 6. You can represent 3^6 as repeated multiplication.

$$\begin{aligned} 3^6 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 729 \end{aligned}$$

C 3 y^x 6 = 729.

You could think of 3^6 as
$(3 \times 3) \times (3 \times 3) \times (3 \times 3)$
$= 9 \times 9 \times 9$
$= 9^3$
or
$(3 \times 3 \times 3) \times (3 \times 3 \times 3)$
$= 27 \times 27$
$= 27^2$
Are there other possibilities?

Show You Know

Evaluate each power.

a) 6^2 **b)** 3^4 **c)** 5^3

Example 3: Powers With Negative Bases

Evaluate each power.

a) $(-2)^4$

b) -2^4

c) $(-4)^3$

d) $-(-5)^6$

Solution

a) In the power $(-2)^4$, the base is -2 and the exponent is 4. The exponent applies to the negative sign because -2 is enclosed in parentheses.

You can write the power as repeated multiplication.

$$\begin{aligned}(-2)^4 &= (-2) \times (-2) \times (-2) \times (-2)\\ &= 16\end{aligned}$$

Why is the answer positive?

b) In the power -2^4, the base is 2 and the exponent is 4. The exponent does not apply to the negative sign because -2^4 is the same as $-(2^4)$.

$$\begin{aligned}-2^4 &= -(2^4)\\ &= -(2 \times 2 \times 2 \times 2)\\ &= -16\end{aligned}$$

c) In the power $(-4)^3$, the base is -4 and the exponent is 3.

$$\begin{aligned}(-4)^3 &= (-4) \times (-4) \times (-4)\\ &= -64\end{aligned}$$

Why is the answer negative?

d) In the expression $-(-5)^6$, the base is -5 and the exponent is 6. The exponent does not apply to the first negative sign because the first negative sign lies outside the parentheses.

$$\begin{aligned}-(-5)^6 &= -[(-5) \times (-5) \times (-5) \times (-5) \times (-5) \times (-5)]\\ &= -(15\ 625)\\ &= -15\ 625\end{aligned}$$

C (5 +/− y^x 6) +/− −15625.

Show You Know

a) Explain how $(-5)^2$ and -5^2 are different and how they are the same.

b) Evaluate $(-6)^2$ and $(-6)^5$.

Key Ideas

- A power is a short way to represent repeated multiplication.
  $7 \times 7 \times 7 = 7^3$
- A power consists of a base and an exponent. The base represents the number you multiply repeatedly. The exponent represents the number of times you multiply the base.

base exponent

$(-3)^5$

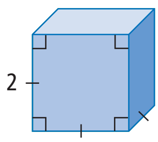
power

Check Your Understanding

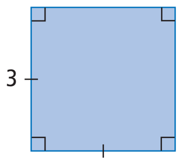
Communicate the Ideas

1. Explain why it is often easier to write an expression as a power rather than as repeated multiplication. Use a specific example.

2. Explain how the two diagrams and calculations show that 2^3 and 3^2 are different.



Volume: $2 \times 2 \times 2 = 8$



Area: $3 \times 3 = 9$

3. Pani says, "When you evaluate a power with a negative base and an even exponent, you get a positive value. When you evaluate a power with a negative base and an odd exponent, you get a negative value." Is Pani correct? Justify your answer.

Practise

For help with #4 and #5, refer to Example 1 on page 93.

4. Write each expression as a power, and evaluate.

a) 7×7

b) $3 \times 3 \times 3$

c) $8 \times 8 \times 8 \times 8 \times 8$

d) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

5. Write each expression as a power. Identify the base and the exponent in each power. Then, evaluate.

a) $1 \times 1 \times 1 \times 1$

b) $2 \times 2 \times 2 \times 2 \times 2$

c) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

d) 13

For help with #6 to #9, refer to Example 2 on page 94.

6. Evaluate each power.

a) 5^2 **b)** 3^3 **c)** 4^5

7. What is the value of each power?

a) 8^3 **b)** 2^6 **c)** 1^9

8. Copy and complete the table.

	Repeated Multiplication	Exponential Form	Value
a)	$6 \times 6 \times 6$	6^3	■
b)	$3 \times 3 \times 3 \times 3$	■	■
c)	■	■	49
d)	■	11^2	■
e)	■	■	125

9. Does $4^3 = 3^4$? Show how you know.

For help with #10 to #13, refer to Example 3 on page 95.

10. Evaluate each power.

a) $(-9)^2$ **b)** -5^3 **c)** $(-2)^7$

11. What is the value of each power?

a) -8^2 **b)** $(-1)^5$ **c)** $-(-3)^7$

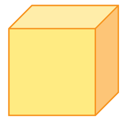
12. Copy and complete the table.

	Repeated Multiplication	Exponential Form	Value
a)	$(-3) \times (-3) \times (-3)$	$(-3)^3$	■
b)	$(-4) \times (-4)$	$(-4)^2$	■
c)	$(-1) \times (-1) \times (-1)$	■	■
d)	■	$(-7)^2$	■
e)	■	■	-1000

13. Does $(-6)^4 = -6^4$? Show how you know.

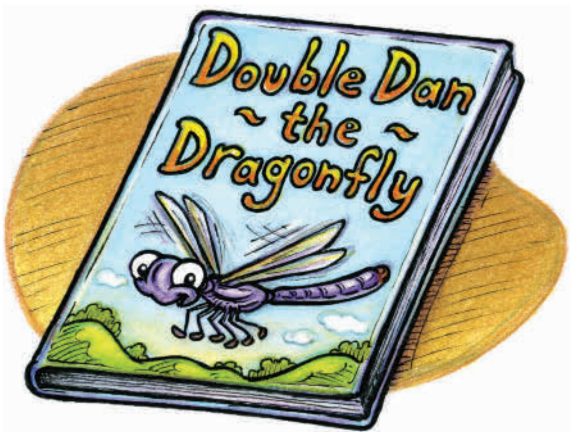
Apply

14. The volume of a cube with an edge length of 3 cm is 27 cm^3. Write the volume in repeated multiplication form and exponential form.



3 cm

15. In a children's story, Double Dan the Dragonfly is growing fast. His body length is doubling every month. At the beginning of the story, his length is 1 cm.



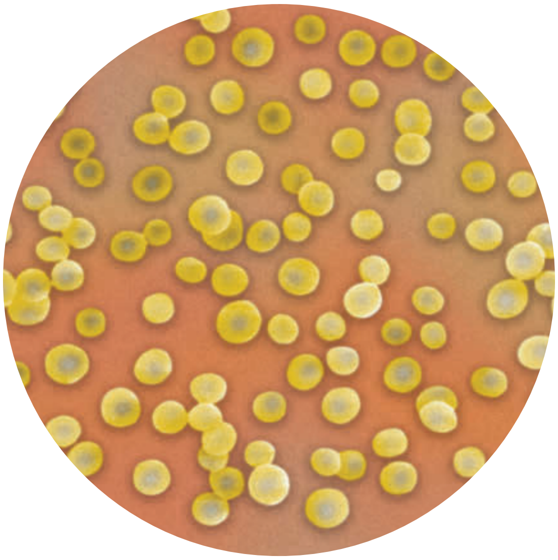
a) Create a table to show how Dan's body length increases every month for ten months.

b) What is his body length five months after the beginning of the story? Express your answer as a power. Then, evaluate.

c) After how many months is his body length more than 50 cm?

16. Arrange the following powers from least to greatest value: 1^{22}, 3^4, 4^3, 2^5, 7^2.

17. A single bacterium doubles in number every hour. How many bacteria are present after 15 h?



18. Express 9 as a power where the exponent is 2 and the base is

a) positive

b) negative

19. Explain what the following statement means using numerical examples:

Multiplication is a way to represent repeated addition, and powers are a way to represent repeated multiplication.

20. The power 7^3 can be read as "seven cubed." Draw a picture of a cube with a volume of 7^3 cubic units, or 343 cubic units. Label appropriate dimensions for the cube.

21. Represent 144 in three different ways using repeated multiplication.

Extend

22. Evaluate the powers of 5 from 5^3 to 5^{10}. Use only whole numbers as exponents.

a) What do you notice about the last three digits of each value?

b) Predict the last three digits if you evaluate 5^{46}.

23. Evaluate the powers of 3 from 3^1 to 3^{12}. Use only whole numbers as exponents.

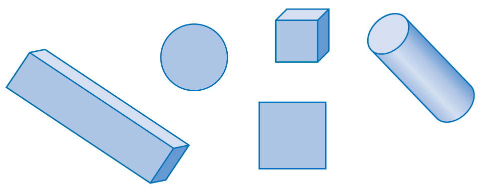
a) What do you notice about the units digit?

b) Predict the units digit if you evaluate 3^{63}. Explain how you arrived at your answer.

Math Link

Some formulas use exponents. Two that you are familiar with are given below.

- $SA = 6s^2$
- $V = \pi r^2 h$



a) Rewrite each formula using repeated multiplication. Identify what the formula represents and how you would use it.

b) For the mobile you will build at the end of the chapter, you will need to use formulas. Identify two formulas that contain exponents, for the shapes shown. Write each formula using repeated multiplication.

3.2 Exponent Laws

Focus on...

After this lesson, you will be able to...

- explain the exponent laws for
  - product of powers
  - quotient of powers
  - power of a power
  - power of a product
  - power of a quotient

The environmental club has permission to use a rectangular plot of land in the school yard for composting and recycling storage. If they know the dimensions of the plot of land, how can they determine the area? If they know the area and the length of the land, how can they determine the width?

Explore Operations on Powers

1. The environmental club learns that the area of the plot of land is 64 m^2.

a) What are the possible whole dimensions of the rectangular plot of land?

b) What is 64 expressed as a power of 2?

c) Show how you can express each of the dimensions in part a) using powers of 2.

2. a) Describe any patterns you observe in your expressions from #1c).

b) Choose a base other than 2. Determine the product of a pair of powers with the base you chose. Does your observation still apply?

3. The environmental club is given another plot of land behind the school to use for a garden. The table gives some possible dimensions and the area of the rectangular plot.

Length of the Rectangular Plot (m)	Width of the Rectangular Plot (m)	Area of the Rectangular Plot (m²)
3^4	3^1	3^5
3^3	3^2	3^5
3^2	3^3	3^5

a) Describe any patterns you observe in the table.

b) Imagine that you are given only the area and length as powers with the same base. Use your patterns to describe how you can determine the width, in exponential form.

What operation do you perform to determine width when you are given the area and length?

c) Choose a base other than 3. Determine the quotient of a pair of powers with the base you chose. Does your observation still apply?

Reflect and Check

4. a) Explain how you can write a product of powers as a single power.

b) Explain how you can write a quotient of powers as a single power.

5. Make up a problem of your own that involves multiplication or division of powers with the same base. Exchange problems with a classmate. Check each other's solution.

Link the Ideas

Example 1: Multiply Powers With the Same Base

Write each product of powers as a single power. Then, evaluate the power.

a) $2^3 \times 2^2$

b) $(-3)^2 \times (-3)^5$

Solution

a) *Method 1: Use Repeated Multiplication*

Rewrite the multiplication statement using repeated multiplication.

$$\begin{aligned} 2^3 \times 2^2 &= (2 \times 2 \times 2) \times (2 \times 2) \\ &= 2^5 \\ &= 32 \end{aligned}$$

There are five factors of 2 in the repeated multiplication.

Method 2: Apply the Exponent Laws

Since the bases are the same, you can add the exponents.

$$\begin{aligned} 2^3 \times 2^2 &= 2^{3+2} \\ &= 2^5 \\ &= 32 \end{aligned}$$

b) Since the bases are the same, you can add the exponents.

$$\begin{aligned} (-3)^2 \times (-3)^5 &= (-3)^{2+5} \\ &= (-3)^7 \\ &= -2187 \end{aligned}$$

Literacy Link

Another term for repeated multiplication is *factored form*.

Literacy Link

Multiplying Powers

When multiplying powers with the same base, add the exponents to write the product as a single power.

$(a^m)(a^n) = a^{m+n}$

Show You Know

Evaluate each expression in two different ways.

a) $4^3 \times 4^5$

b) $(-5)^2 \times (-5)^3$

Did You Know?

Some common viruses require at least 2^{87} viral particles in the human body before symptoms occur.

Digital rights not available.

Example 2: Divide Powers With the Same Base

Write each quotient as a single power. Then, evaluate the power.

a) $2^6 \div 2^2$

b) $(-5)^9 \div (-5)^6$

Solution

a) *Method 1: Use Repeated Multiplication*

Rewrite each power using repeated multiplication.

$$\begin{aligned} 2^6 \div 2^2 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2) \\ &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} \\ &= \frac{2 \times 2 \times 2 \times 2 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}} \\ &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \\ &= 16 \end{aligned}$$

You can divide the common factors in the numerator and denominator.

Method 2: Apply the Exponent Laws

Since the bases are the same, you can subtract the exponents.

$$\begin{aligned} 2^6 \div 2^2 &= 2^{6-2} \\ &= 2^4 \\ &= 16 \end{aligned}$$

b) Since the bases are the same, you can subtract the exponents.

$$\begin{aligned} (-5)^9 \div (-5)^6 &= (-5)^{9-6} \\ &= (-5)^3 \\ &= -125 \end{aligned}$$

Literacy Link

Dividing Powers

When dividing powers with the same base, subtract the exponents to write the quotient as a single power.

$a^m \div a^n = a^{m-n}$

Show You Know

Evaluate each expression in two different ways.

a) $2^5 \div 2^4$

b) $(-3)^{10} \div (-3)^7$

Example 3: Raise Powers, Products, and Quotients to an Exponent

a) Write the expression $(2^3)^2$ as a single power. Then, evaluate.

b) Write the expression $[2 \times (-3)]^4$ as the product of two powers. Then, evaluate.

c) Write the expression $\left(\frac{3}{4}\right)^3$ as the quotient of two powers. Then, evaluate.

Solution

a) *Method 1: Use Repeated Multiplication*

$$\begin{aligned}(2^3)^2 &= 2^3 \times 2^3\\ &= (2 \times 2 \times 2) \times (2 \times 2 \times 2)\\ &= 2^6\\ &= 64\end{aligned}$$

How many factors of 2 are there in the repeated multiplication?

Method 2: Apply the Exponent Laws

You can multiply the exponents.

$$\begin{aligned}(2^3)^2 &= 2^6\\ &= 64\end{aligned}$$

b) *Method 1: Use Repeated Multiplication*

$$\begin{aligned}[2 \times (-3)]^4 &= [2 \times (-3)] \times [2 \times (-3)] \times [2 \times (-3)] \times [2 \times (-3)]\\ &= 2 \times 2 \times 2 \times 2 \times (-3) \times (-3) \times (-3) \times (-3)\\ &= 2^4 \times (-3)^4\\ &= 16 \times 81\\ &= 1296\end{aligned}$$

How do you know that you can rearrange the factors?

Method 2: Apply the Exponent Laws

You can write each factor in the product with the same exponent.

$$\begin{aligned}[2 \times (-3)]^4 &= 2^4 \times (-3)^4\\ &= 16 \times 81\\ &= 1296\end{aligned}$$

How else could you evaluate this expression?

c) *Method 1: Use Repeated Multiplication*

$$\begin{aligned}\left(\frac{3}{4}\right)^3 &= \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)\\ &= \frac{3 \times 3 \times 3}{4 \times 4 \times 4}\\ &= \frac{3^3}{4^3}\\ &= \frac{27}{64}\end{aligned}$$

Why can you not divide these powers by subtracting the exponents?

Method 2: Apply the Exponent Laws

You can write each number in the quotient with the same exponent.

$$\begin{aligned}\left(\frac{3}{4}\right)^3 &= \frac{3^3}{4^3}\\ &= \frac{27}{64}\end{aligned}$$

Literacy Link

Raising a Power to an Exponent

When a power is raised to an exponent, multiply the exponents to write the expression with a single exponent.

$(a^m)^n = a^{mn}$

Literacy Link

Raising a Product to an Exponent

When a product is raised to an exponent, you can rewrite each factor in the product with the same exponent.

$(ab)^m = a^m b^m$

Literacy Link

Raising a Quotient to an Exponent

When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

Show You Know

a) Write $[(-3)^4]^3$ as a single power. Then, evaluate.

b) Write $(5 \times 4)^2$ as the product of two powers. Then, evaluate.

c) Write $\left(\frac{2}{5}\right)^5$ as the quotient of two powers. Then, evaluate.

Example 4: Evaluate Quanities With an Exponent of Zero

Evaluate 3^0.

Strategies

Make a Table

Look for a Pattern

Solution

You can use a table to determine a pattern in the powers of 3.

Power	Value
3^4	81
3^3	27
3^2	9
3^1	3
3^0	■

Determine the pattern in the values.
$81 \div 3 = 27$
$27 \div 3 = 9$
$9 \div 3 = 3$

Each value can be found by dividing the value above it by 3.
$3 \div 3 = 1$

So, the value of 1 belongs in the blank.
$3^0 = 1$

Check:
You can use division to show that $3^0 = 1$.
Choose any power of 3, such as 3^4. Divide it by itself.

$$\frac{3^4}{3^4} = 3^{4-4} = 3^0 \qquad \frac{3^4}{3^4} = \frac{\cancel{3^4}^{\,1}}{\cancel{3^4}_{\,1}} = 1$$

So, $3^0 = 1$.

You can also check using a calculator.

C 3 y^x 0 = 1.

Literacy Link

Raising a Quantity to an Exponent of Zero

When the exponent of a power is 0, the value of the power is 1 if the base is not equal to 0.

$a^0 = 1, a \neq 0$

Show You Know

Evaluate each expression.

a) $(-5)^0$ **b)** -5^0

c) $-(5)^0$ **d)** 5^0

Key Ideas

- You can apply the exponent laws to help simplify expressions.
  - You can simplify a product of powers with the same base by adding exponents.
    $a^m \times a^n = a^{m+n}$ $\quad$ $3^7 \times 3^2 = 3^{7+2} = 3^9$
  - You can simplify a quotient of powers with the same base by subtracting the exponents.
    $a^m \div a^n = a^{m-n}$ $\quad$ $5^8 \div 5^2 = 5^{8-2} = 5^6$
  - You can simplify a power that is raised to an exponent by multiplying the two exponents.
    $(a^m)^n = a^{mn}$ $\quad$ $(3^4)^5 = 3^{4\times 5} = 3^{20}$
  - When a product is raised to an exponent, you can rewrite each number in the product with the same exponent.
    $(a \times b)^m = a^m \times b^m$ $\quad$ $(5 \times 6)^3 = 5^3 \times 6^3$
  - When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.
    $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad$ $\left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}$
  - When the exponent of a power is 0, the value of the power is 1 if the base is not equal to 0.
    $a^0 = 1, a \neq 0$ $\quad$ $(-10)^0 = 1$

Check Your Understanding

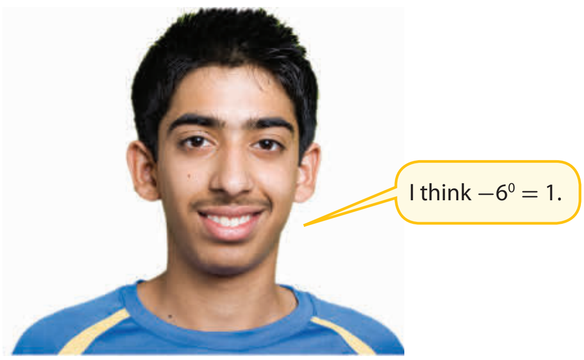
Communicate the Ideas

1. Explain why $(4^2)^5 = 4^{10}$.

2. Show whether the expression $(-2)^2 \times (-2)^3$ and the expression $[(-2)^2]^3$ are equal.

3. Explain why $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$.

4. Is Ranbir correct? Justify your answer.



Practise

For help with #5 to #8, refer to Example 1 on page 101.

5. Write each expression as a single power. Then, evaluate each power.

a) $4^3 \times 4^4$

b) $7^2 \times 7^4$

c) $(-3)^5 \times (-3)^2$

6. Rewrite each expression as a single power. Then, evaluate.

a) $5^2 \times 5^3$

b) $(-6)^3 \times (-6)^3$

c) $8^1 \times 8^2$

7. Write each expression as a product of two powers, and then as a single power.

a) $(4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4)$

b) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$

c) $(9 \times 9 \times 9 \times 9) \times (9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9)$

8. Write the following expression in repeated multiplication form, and then as a single power: $3^2 \times 3^3 \times 3^4$.

For help with #9 to #13, refer to Example 2 on page 102.

9. Write each expression as a single power. Then, evaluate each power.

a) $5^5 \div 5^3$

b) $3^8 \div 3^4$

c) $(-4)^6 \div (-4)^2$

10. Rewrite each expression as a single power. Then, evaluate.

a) $7^4 \div 7^1$

b) $(-8)^8 \div (-8)^6$

c) $(-2)^6 \div (-2)^5$

11. Write each expression as a quotient of two powers, and then as a single power.

a) $(6 \times 6 \times 6 \times 6) \div (6 \times 6 \times 6)$

b) $\dfrac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$

c) $(5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5) \div (5)$

12. Write the following expression as the division of two powers: $(-5)^{7-2}$.

13. Write the numerator and the denominator in exponential form, and then write the expression as a single power:

$$\frac{(6 \times 6) \times (6 \times 6) \times (6 \times 6)}{6 \times 6 \times 6}.$$

For help with #14 to #17, refer to Example 3 on page 102–103.

14. a) Write $(3^2)^4$ as a single power. Evaluate.

b) Write $[7 \times (-3)]^4$ as the product of two powers. Evaluate.

c) Write $\left(\frac{5}{6}\right)^4$ as the quotient of two powers. Evaluate.

15. a) Write $(-4^2)^3$ with one exponent. Evaluate.

b) Write $(3 \times 4)^4$ as the multiplication of two powers. Evaluate.

c) Write $\left(\frac{4}{5}\right)^3$ as the division of two powers. Evaluate.

16. Write the following expression as a power raised to an exponent.

$(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

17. Copy and complete the table.

Expression	Repeated Multiplication	Powers
a) $[2 \times (-5)]^3$	$\blacksquare$	$\blacksquare$
b) $\blacksquare$	$(9 \times 8) \times (9 \times 8)$	$9^2 \times 8^2$
c) $\left(\frac{2}{3}\right)^4$	$\blacksquare$	$\blacksquare$

For help with #18 and #19 refer to Example 4 on page 104.

18. a) Evaluate 2^0. Use a pattern to justify your answer.

b) Check your answer a different way.

19. a) Evaluate -4^0. Show your thinking.

b) Evaluate $(-4^0) \times (-4^0) \times (-4^0)$.

Apply

20. Jake's older model computer has a processing speed of 20^2 MHz. Hanna's new computer has a processing speed of 50^2 MHz.

a) What is the ratio, in fraction form, of Jake's computer processing speed to Hanna's computer processing speed? Do not simplify your answer.

b) Write this ratio as a single power in simplest form.

Did You Know?

According to Moore's Law, computer processing power tends to increase exponentially, doubling every two years. The observation was originally made in 1965 by Gordon Moore, co-founder of Intel. This exponential growth is expected to continue for at least another ten years.

21. Express each of the following as a single power.

a) $(3^2)^4 \times 3^3$

b) $\dfrac{(-4)^2(-4)^4}{(-4)^3}$

22. Jenny was asked to complete the following exercise.

Write the expression as a product of two powers, and then express as a single power: $(7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$.

Find and explain the mistake Jenny made in her solution.

$(7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$
$= 7^5 \times 7^3$
$= 7^{5 \times 3}$
$= 7^{15}$

23. Write three different products. Each product must be made up of two powers and must be equal to 4^5.

Extend

24. Find two different whole numbers that can be placed in the boxes so that the following statement is true.

$0.1 \le \left(\dfrac{\blacksquare}{\blacksquare}\right)^4 \le 0.2$

25. a) Find a pair of whole numbers, excluding zero, that can be placed in the boxes to make the following equation true.
$81^{\blacksquare} = 27^{\blacksquare}$

b) What is a second pair?

26. If $3^x = 11$, use exponent laws to evaluate the following expressions.

a) 3^{2x}

b) $3^{(x + 1)}$

3.3 Order of Operations

Focus on...

After this lesson, you will be able to...
- use the order of operations on expressions with powers
- apply the laws of exponents

In the game show Power of 5, contestants try to answer eight questions in their pursuit of $10 million.

Explore Order of Operations With Powers

1. How many times greater is each prize value than the previous prize value? Explain how you arrived at your answer.

2. a) What is an expression in exponential form that represents the prize value for answering the fourth question correctly? Compare your answer with a classmate's.

b) How could you find the value of this expression?

3. Write expressions in exponential form for the top prize value and for the prize value for answering the fifth question correctly. Use these expressions to write an expression that shows the difference between these prize values. Then, evaluate the expression. Compare your answer with a classmate's.

Reflect and Check

4. a) Identify the coefficient and the power in the expression 128×5^7.

The expression 128×5^7 can also be written as $128(5^7)$.

b) What does each of these values represent in the Power of 5 game show?

5. a) What does the expression $128(5^7) - 128(5^3)$ represent in terms of prize values in the Power of 5 game show?

b) Describe the steps you would use to evaluate this expression.

Literacy Link

A *coefficient* is a number that multiplies an expression. In $-5(4)^2$, the coefficient is -5.

Link the Ideas

Example 1: Determine the Product of a Power

Evaluate.

a) $3(2)^4$ **b)** $-3(-5)^2$ **c)** -4^4

Solution

a) *Method 1: Use Repeated Multiplication*

You can use repeated multiplication for the power.

$$\begin{aligned}3(2)^4 &= 3 \times 2^4\\ &= 3 \times 2 \times 2 \times 2 \times 2\\ &= 48\end{aligned}$$

Method 2: Use Order of Operations

$$\begin{aligned}3 \times 2^4 &= 3 \times 16\\ &= 48\end{aligned}$$

Evaluate the power first.

Method 3: Use a Calculator

C 3 × 2 y^x 4 = 48.

b) $$\begin{aligned}-3(-5)^2 &= -3(25)\\ &= -75\end{aligned}$$

c) $$\begin{aligned}-4^4 &= -1 \times 4^4\\ &= -1 \times 256\\ &= -256\end{aligned}$$

Literacy Link

The order of operations is:
- brackets
- exponents (powers)
- divide and multiply in order from left to right
- add and subtract in order from left to right

Tech Link

The key sequence may be different on different calculators. Experiment with performing order of operations on your calculator. Record the proper key sequence for your calculator.

Show You Know

Evaluate. Use a calculator to check your answer.

a) 4×3^2

b) $6(-2)^3$

c) -7^2

Example 2: Evaluate Expressions With Powers

Evaluate.

a) $4^2 - 8 \div 2 + (-3^2)$ **b)** $-2(-15 - 4^2) + 4(2 + 3)^3$

Solution

a) *Method 1: Use Order of Operations*

$$\begin{aligned} &4^2 - 8 \div 2 + (-3^2) \\ &= 16 - 8 \div 2 + (-9) \\ &= 16 - 4 + (-9) \\ &= 12 + (-9) \\ &= 3 \end{aligned}$$

Method 2: Use a Calculator

C 4 y^x 2 − 8 ÷ 2 + 3 +/− y^x 2 = 3.

You may need to use a different key sequence on your calculator.

b) *Method 1: Use Order of Operations*

$$\begin{aligned} &-2(-15 - 4^2) + 4(2 + 3)^3 \\ &= -2(-15 - 16) + 4(5)^3 \\ &= -2(-31) + 4(5)^3 \\ &= -2(-31) + 4(125) \\ &= 62 + 4(125) \\ &= 62 + 500 \\ &= 562 \end{aligned}$$

Method 2: Use a Calculator

C 2 +/− × (15 +/− − 4 x^2) + 4 ×
(2 + 3) y^x 3 = 562.

Show You Know

Evaluate.

a) $4^2 + (-4^2)$ **b)** $8(5 + 2)^2 - 12 \div 2^2$

Key Ideas

- Expressions with powers can have a numerical coefficient. Evaluate the power, and then multiply by the coefficient.

Expression	Coefficient	Power	Repeated Multiplication	Value
$5(4^2)$	5	4^2	$5 \times 4 \times 4$	80
$(-2)^4$	1	$(-2)^4$	$(-2)(-2)(-2)(-2)$	16
-3^4	−1	3^4	$-1 \times 3 \times 3 \times 3 \times 3$	−81

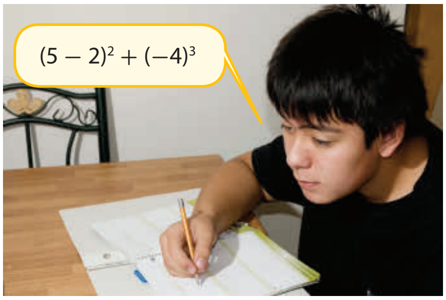
- Evaluate expressions with powers using the proper order of operations.
  - brackets
  - exponents
  - divide and multiply in order from left to right
  - add and subtract in order from left to right

Check Your Understanding

Communicate the Ideas

1. Using the terms *coefficient* and *base*, explain why the two expressions -2^2 and $(-2)^2$ are different and result in different answers.

2. Your classmate, Han, needs help with his homework. Explain how to evaluate $(5 - 2)^2 + (-4)^3$.



3. Identify the incorrect step in the following solution. Show how to correct it. What is the correct answer?

$(3 + 5)^2 - 4 \times 3^2$	
$= 8^2 - 4 \times 3^2$	Step 1
$= 64 - 4 \times 3^2$	Step 2
$= 60 \times 3^2$	Step 3
$= 60 \times 9$	Step 4
$= 540$	Step 5

4. Maria was asked to evaluate 128×5^3. What mistake did Maria make in her solution?

128×5^3
$= 640^3$
$= 262\,144\,000$

Practise

For help with #5 to #7, refer to Example 1 on page 109.

5. Evaluate each expression.

a) $4(2)^5$ **b)** $7(-3)^2$
c) $-2(5^4)$ **d)** $3(-2^2)$

6. Write each expression using a coefficient and a power. Then, find the value of each expression.

a) $4 \times 2 \times 2 \times 2 \times 2$
b) $3 \times (-2) \times (-2) \times (-2)$
c) $7(10)(10)(10)(10)(10)$
d) $-1 \times 9 \times 9 \times 9 \times 9$

7. Write the key sequence you would use to evaluate each expression using your calculator. What is the answer?

a) 4×3^2
b) $-5(4)^3$

For help with #8 and #9, refer to Example 2 on page 110.

8. Evaluate.

a) $3^2 + 3^2$
b) $(2 + 7)^2 - 11$
c) $7^3 - 3(-4)^3$
d) $9 + (-2)^3 - 2(-6^2)$

9. Find the value of each expression.

a) $7 - 2(3^2)$

b) $(-4 - 3)^2 + (-3)^2$

c) $(-2)^6 \div 4^3$

d) $24 - 2^2 + (7^2 - 5^2)$

Apply

10. For each pair of expressions, which one has a greater value? How much greater is it?

a)	$3(2)^3$	$2(3)^2$
b)	$(3 \times 4)^2$	$3^2 \times 4^2$
c)	$6^3 + 6^3$	$(6 + 6)^3$

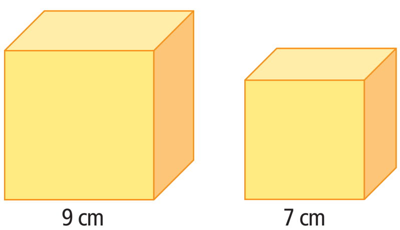
11. Find the step where Justin made an error. Show the correct answer.

$(-3 + 6)^2 - 4 \times 3^2$	
$= 3^2 - 4 \times 3^2$	Step 1
$= 9 - 4 \times 9$	Step 2
$= 5 \times 9$	Step 3
$= 45$	Step 4

12. Find the step where Katarina made an error. What is the correct answer?

$32 \div (-2)^3 + 5(4)^2$	
$= 32 \div (-8) + 5 \times 8$	Step 1
$= -4 + 5 \times 8$	Step 2
$= -4 + 40$	Step 3
$= 36$	Step 4

13. Write an expression with powers to determine the difference between the volume of the small cube and the volume of the large cube. What is the difference?



14. Read the following riddle and then answer the questions below.

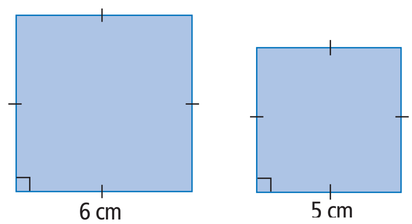
In downtown Victoria, there are seven pink houses. Every pink house has seven pink rooms, every pink room has seven cats, and every cat has seven kittens.

a) How many pink rooms are there?

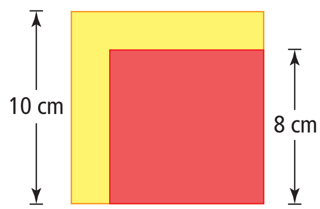
b) How many kittens are there?

c) Write an expression using powers of 7 to determine the total number of houses, rooms, cats, and kittens. Evaluate your expression.

15. Write an expression with powers to determine the difference between the area of the large square and the area of the small square. What is the difference?



16. A red square with a side length of 8 cm is placed on a yellow square with a side length of 10 cm. Write an expression with powers to determine the visible yellow area. What is the visible yellow area?



Extend

17. What is the value of 5^{3^2}?

18. In a game show called The Pyramid of Money, a contestant must successfully answer a set of questions. The first question is worth \$3125. Each question after that is worth four times the value of the previous question.

a) What is the value of question 2? question 3? question 4?

b) How many questions would a contestant have to answer correctly before becoming eligible to answer a question with a value of \$3 200 000?

c) Which question is represented by the expression 3125×4^7?

d) Write an expression with powers that represents the sum of the values of the first four questions.

19. A phone tree is used to notify the players on a football team about a change in the time for their next game. Each of the three coaches calls two different players, and then each player calls two more players. Each person only makes two calls. The chart shows the number of people calling and receiving calls for each round of two calls.

Round of Calls	Number of People Calling	Number of Calls Received	Total Number of People Notified
First	3	6	$3 + 6 = 9$
Second	6	12	$9 + 12 = 21$
Third	12	24	$21 + 24 = 45$
Fourth	24	■	$45 +$ ■ $= 93$

a) What value belongs in each unknown box?

b) Write an expression for the number from part a) as a product of 3 and a power of 2.

c) What does the 3 in part b) represent?

d) What does the exponent in part b) represent?

e) Imagine that the phoning continued. Determine an expression for the number of calls received in the sixth round and evaluate it.

f) If five coaches started the phone tree instead of three, what would be the number of calls received in the third round?

20. Use four 2s to write an expression with the greatest possible value.

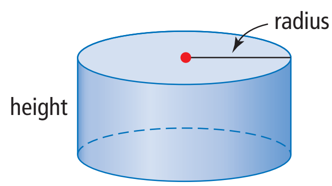
Math Link

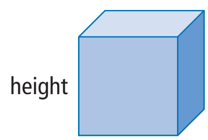
You are planning to build a mobile with a cylinder and a cube.

a) The height and radius of the cylinder and the height of the cube will all be the same measurement. Choose a whole number measurement, in centimetres.

b) Write an expression in exponential form to calculate the difference in the area of material required to make each shape. Which shape requires more material? How much more? Express your answer to the nearest tenth of a square centimetre.

c) Write an expression in exponential form to calculate the total area of material needed to make both shapes. Express your answer to the nearest tenth of a square centimetre.





3.4 Using Exponents to Solve Problems

Focus on...

After this lesson, you will be able to...

- solve problems that require combining powers
- use powers to solve problems that involve repeated multiplication

Near Alberta/British Columbia border

The mountain pine beetle has the ability to double its population in one year if conditions are right. These beetles live in mature pine trees by boring into the bark. Only 6 mm long, these small beetles can kill pine trees if their numbers are great enough.

The mountain pine beetle has destroyed thousands of pine trees in British Columbia and Alberta. It is estimated that by 2013, 80% of the pine trees in BC will be eliminated by the mountain pine beetle.

Explore Operations on Powers

A population of 10 000 mountain pine beetles doubles each year.

1. Create a table to show the growth of the population of pine beetles over three years.

2. Express the population each year as a product of 10 000 and a power of 2. Add this information to your table.

3. What patterns do you notice in your table?

4. How could you determine the number of beetles in ten years without extending the table? Show two methods.

5. How would your table be different if the beetles tripled in number each year?

Reflect and Check

6. Write an expression in exponential form to determine the number of beetles in n years. Explain what each value represents.

7. How could you determine the number of years that have passed if the number of beetles is 640 000?

WWW Web Link

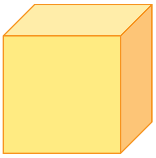
To read more about the mountain pine beetle, go to www.mathlinks9.ca and follow the links.

Link the Ideas

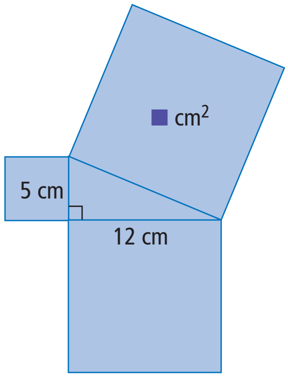
Example 1: Use Formulas to Solve Problems

Write an exponential expression to solve each problem.

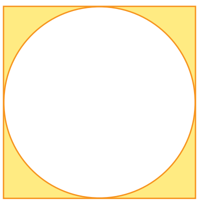
a) What is the surface area of a cube with an edge length of 4 cm?



b) Find the area of the square attached to the hypotenuse in the diagram.



c) A circle is inscribed in a square with a side length of 20 cm. What is the area of the shaded region?



Solution

a) The formula for the surface area of a cube is $SA = 6s^2$, where s is the edge length of the cube.

$SA = 6s^2$
$SA = 6(4)^2$
$SA = 6(16)$
$SA = 96$

The surface area of the cube is 96 cm².

b) The diagram models the Pythagorean relationship. The relationship between the areas of the squares on the sides of a right triangle is represented by the formula $c^2 = a^2 + b^2$, where a and b are the legs of the triangle and c is the hypotenuse.

$c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c^2 = 169$

The area of the square attached to the hypotenuse is 169 cm².

c) *Method 1: Calculate in Stages*

The formula for the area of a square is $A = s^2$, where s is the side length of the square.

$A = s^2$

$A = 20^2$

$A = 400$

The area of the square is 400 cm^2.

The formula for the area of a circle is $A = \pi r^2$, where r is the radius of the circle.

The diameter of the circle is 20 cm. Therefore, the radius is 10 cm.

$A = \pi r^2$

$A = \pi(10)^2$

$A = \pi(100)$

$A \approx 314 \ldots$

The area of the circle is approximately 314 cm^2.

Calculate the area of the shaded region. You can subtract the area of the circle from the area of the square.

$400 - 314 = 86$

The area of the shaded region is about 86 cm^2.

Method 2: Evaluate One Expression

Calculate the area of the shaded region. You can subtract the area of the circle from the area of the square.

$A = s^2 - \pi r^2$

$A = 20^2 - \pi(10)^2$

$A = 400 - \pi(100)$

$A \approx 400 - 314$

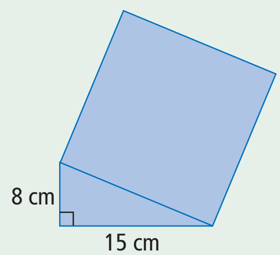
$A \approx 86$

The area of the shaded region is about 86 cm^2.

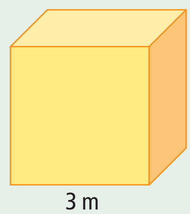
Show You Know

Use a formula to solve each problem.

a) A right triangle has two shorter sides that measure 8 cm and 15 cm. What is the area of a square attached to the hypotenuse of the right triangle?



b) What is the surface area of a cube with an edge length of 3 m?



Example 2: Develop a Formula to Solve a Problem

A dish holds 100 bacteria. It is known that the bacteria double in number every hour. How many bacteria will be present after each number of hours?

a) 1 **b)** 5 **c)** n

Solution

a) After 1 h, the bacteria population doubles.
$100 \times 2 = 200$
After 1 h, there will be 200 bacteria.

b) In a period of 5 h, the bacteria population doubles five times.

$$\begin{aligned} & 100 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 100(2^5) \\ &= 100(32) \\ &= 3200 \end{aligned}$$

After 5 h, there will be 3200 bacteria.

c) After n hours, the bacteria population doubles n times.
Number of bacteria $= 100(2^n)$
After n hours, there will be $100(2^n)$ bacteria.

Strategies
Use a Variable

Show You Know

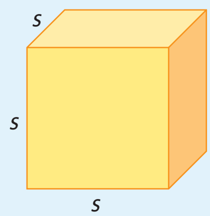
A type of bacterium is known to triple every hour. There are 50 bacteria to start with. How many will there be after each number of hours?

a) 3 **b)** 5 **c)** t

Key Ideas

- Powers are found in many formulas. When repeated multiplication is present in a formula, it is represented as a power. The use of powers keeps the formula as short as possible.
- Many patterns that involve repeated multiplication can be modelled with expressions that contain powers.

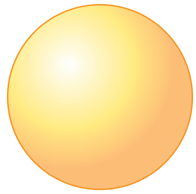
Volume of a cube $= s^3$



Check Your Understanding

Communicate the Ideas

1. The surface area, SA, of a sphere can be calculated using the formula $SA = 4 \times \pi \times r \times r$, where r is the radius. Rewrite the formula using powers and no multiplication signs. Identify the coefficient, variable, and exponent in your formula.

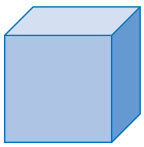


2. Explain what each number and letter represents in the following expression: Number of bacteria $= 100(2)^n$.

Practise

For help with #3 and #4, refer to Example 1 on page 115.

3. What is the volume of a cube with an edge length of 6 cm? Write an exponential expression to solve the problem.



6 cm

4. Which is larger, the area of a square with a side length of 14 cm or the surface area of a cube with an edge length of 6 cm? Show your work. Write an exponential expression to solve the problem.

For help with #5 and #6, refer to Example 2 on page 117.

5. A colony of bacteria triples every hour. If there are 20 bacteria now, how many will there be after each amount of time?
   a) 1 h
   b) 6 h
   c) n hours

6. A population of 200 bacteria has the perfect conditions to double every 20 min. How many bacteria will there be after each amount of time?
   a) 20 min
   b) 60 min
   c) 4 h

Apply

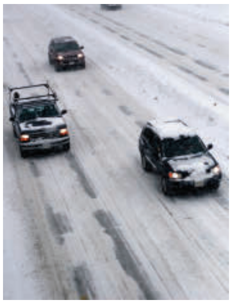
7. Sarah wants to send a cube-shaped package to her cousins in Pond Inlet, Nunavut. She is going to wrap the package in brown paper The side length of the package measures 46 cm. What is the minimum amount of paper Sarah will need?

8. Albert Einstein stated that $E = mc^2$. The mass, m, is measured in kilograms, and the speed of light, c, is approximately 300 000 000 m/s. The amount of energy released, E, is measured in joules. How much energy is released if 1 g of mass is converted to energy?

9. A true/false quiz with four questions has a total of 2^4 possible sets of answers. One possible set of answers is TFFF.
   a) What does the exponent, 4, represent?
   b) What does the base, 2, represent?
   c) List the remaining sets of possible answers.
   d) How many sets of answers are possible for a quiz with ten true/false questions? Express as a power and then evaluate.

10. A standard combination lock has 60 numbers on its face from 0 to 59. A combination consists of three numbers, and numbers can be repeated. What is the total number of possible combinations? Express as a power and then evaluate.

11. A formula that estimates the stopping distance for a car on an icy road is $d = 0.75s\left(\frac{c}{1000}\right)^2$. The distance, d, is measured in metres. The speed of the car, s, is in kilometres per hour. The mass of the car, c, is in kilograms.



a) What is the stopping distance for a 1000-kg car travelling at 50 km/h?

b) What is the stopping distance for a 2000-kg car travelling at 60 km/h?

c) Write the key sequence for solving part b) using your calculator.

Extend

12. The number 10^{100} is known as a googol.

a) Research where the term googol originated. Why do you think the founders of Google™ used that name for their search engine?

b) How many zeros would follow the 1 if you wrote 10^{100} as a whole number?

c) If you were able to continue writing zeros without stopping, how long would it take you to write a googol as a whole number? Explain why you believe your answer is correct.

13. Sometimes powers have the same bases and different exponents. At other times, powers have the same exponents and different bases.

a) Use the examples below to explain how you would compare such powers. Generate rules you could use for comparing these powers.

3^5 and 3^7

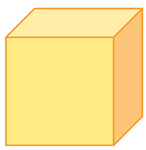
6^4 and 5^4

b) How could you use the exponent laws to alter these powers so you could easily compare them without finding their values?

32^4 and 64^3

Math Link

Stefan plans to make a mobile out of five equal-sized cubes with a side length of 3 cm. He has the exact amount of paper he needs to create his mobile. However, he then decides he would like the cubes to be larger.


3 cm

a) Suggest a larger size for the cubes. Use whole number dimensions.

b) What is the minimum area of paper Stefan will need to create the five cubes that you suggest? Give your answer as an expression in exponential form. Then, evaluate.

c) How much more paper will Stefan need than the original amount he had? Give your answer as an expression in exponential form. Then, evaluate.

Chapter 3 Review

Key Words

For #1 to #5, use the clues to unscramble the letters.

1. T F N F E E I C C O I

a number that multiplies a power

2. N N T O E I P A X L E   M O R F

the form for writing a number so that it is made up of a base and an exponent (two words)

3. E A S B

the number in a power that is multiplied repeatedly

4. W R O E P

an expression made up of a base and an exponent

5. T O X E N P N E

the number in a power that indicates how many times to repeatedly multiply the base by itself

3.1 Using Exponents to Describe Numbers, pages 92–98

6. Write each expression as a power.

a) $2 \times 2 \times 2$

b) $(-3) \times (-3) \times (-3) \times (-3)$

7. Write each power in repeated multiplication form.

a) 4^6

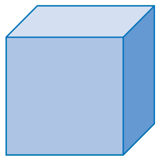
b) 6^4

c) $(-5)^7$

d) -5^7

8. The area of a square on grid paper is 5^2. Evaluate the area. Draw the square and label its area and side length.

9. A cube has an edge length of 4 cm. Express its volume in repeated multiplication form and in exponential form. Then, evaluate.



10. Arrange the following numbers in ascending order:

$4^3 \quad 7^2 \quad -3^4 \quad 9 \quad 2^5$

3.2 Exponent Laws, pages 99–107

11. Rewrite each power in the following products in repeated multiplication form.

a) $3^2 \times 5^4$

b) $(-3)^3 \times 2^6$

12. Write each expression in parentheses as a power. Then, write the entire expression as a single power.

a) $(2 \times 2 \times 2) \times (2 \times 2)$

b) $\dfrac{(4 \times 4)(4 \times 4 \times 4 \times 4)}{(4 \times 4 \times 4)}$

13. Write each expression in repeated multiplication form, and then as a single power.

a) $(-5)^2 \times (-5)^5$

b) $(3^2)^4$

14. Write each expression as the multiplication of two powers.

a) $(6 \times 4)^3$

b) $[(7 \times (-2)]^5$

15. Write each expression as the division of two powers.

a) $\left(\frac{4}{5}\right)^2$

b) $\left(\frac{2}{7}\right)^4$

16. Evaluate.

a) -4^2

b) $(-10)^0$

c) $3^2 \times 3^3$

3.3 Order of Operations, pages 108–113

17. Write the calculator key sequence you would use to evaluate each expression.

a) $(-2)^2 + (-2)^3$

b) $(2^3)^2 - 4 \times 6^0$

c) $(-3)^4 - (-3)^3 + (2 \times 4)^2$

18. Evaluate.

a) $7^2 + (-2)^3 \div (-2)^2$

b) $(2 - 5)^3 + 6^2$

c) $\dfrac{(2)^6(2)^2 - 13 \times 2^0}{(-1 + 2^2)^5}$

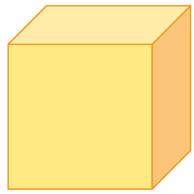
d) $(-1)^{10} + (-22)^0 - \left(\frac{3}{5}\right)^2$

19. Explain the mistake in Ang's solution. Determine the correct answer.

$(-3)^4 + 7 \times 2^3$
$= 81 + 7 \times 8$
$= 88 \times 8$
$= 704$

3.4 Using Exponents to Solve Problems, pages 114–119

20. What is the surface area of a cube with an edge length of 5 m?



21. A population of ten bacteria doubles every hour. This growth can be represented by $N = 10(2)^t$, where N is the number of bacteria, and t is the amount of time, in hours. How many bacteria will there be after each number of hours?

a) 3

b) 6

22. A formula that approximates the distance an object falls through air in relation to time is $d = 4.9t^2$. The distance, d, is measured in metres, and the time, t, in seconds. A pebble breaks loose from a cliff. What distance would it fall in each number of seconds?

a) 1

b) 2

c) 6

Chapter 3 Practice Test

For #1 to #6, choose the best answer.

1. What is the value 3 in the power 4^3 called?

A base **B** power
C exponent **D** coefficient

2. What is the coefficient in the expression $-(-3)^5$?

A -3 **B** -1
C 1 **D** 3

3. What expression is represented by $(3^2)^4$?

A $(3 \times 3)(3 \times 3 \times 3 \times 3)$
B $(3 \times 3 \times 3 \times 3 \times 3 \times 3)$
C $(3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3)$
D $(3 \times 3 \times 3 \times 3)(3 \times 3 \times 3 \times 3)(3 \times 3 \times 3 \times 3)(3 \times 3 \times 3 \times 3)$

4. What expression is equivalent to $(5 \times 4)^2$?

A 10×8 **B** 5×4^2
C $5^2 \times 4$ **D** $5^2 \times 4^2$

5. What is $\frac{(-7)^3(-7)^5}{(-7)^2}$ expressed as a single power?

A $(-7)^6$ **B** $(-7)^{10}$
C $(-7)^{13}$ **D** $(-7)^{17}$

6. Evaluate $(7 - 2)^3 + 48 \div (-2)^4$.

A 338 **B** 128
C 10.8125 **D** -10.8125

Complete the statements in #7 and #8.

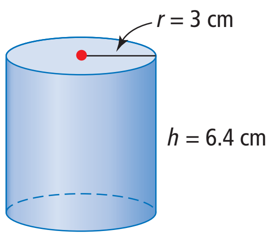
7. The expression $10^5 \times 5^5$ written with only one exponent is ■.

8. The expression $\frac{5^6}{8^6}$ written with only one exponent is ■.

Short Answer

9. Write the expression $\frac{4^4 \times 4}{4^2}$ in repeated multiplication form, and then evaluate.

10. The formula for the volume of a cylinder is $V = \pi r^2 h$. Find the volume, V, of a cylinder with a radius of 3 cm and a height of 6.4 cm. Express your answer to the nearest tenth of a cubic centimetre.



11. A skydiver free falls before opening the parachute. What distance would the skydiver fall during 7 s of free fall? Use the formula $d = 4.9t^2$, where d is distance, in metres, and t is time, in seconds.

12. Write the calculator key sequence you would use to evaluate each expression. Then, evaluate.

a) $(1 - 3)^4 \div 4$
b) $(-2)^0 + 4 \times 17^0$
c) $16 - 9(2^3) + (-4)^2$

13. The prime factorization of 243 is $3 \times 3 \times 3 \times 3 \times 3$. Write 243 as the product of two powers of 3 in as many ways as possible.

14. A formula for estimating the volume of wood in a tree is $V = 0.05hc^2$. The volume, V, is measured in cubic metres. The height, h, and the trunk circumference, c, are in metres. What is the volume of wood in a tree with a trunk circumference of 2.3 m and a height of 32 m? Express your answer to the nearest tenth of a cubic metre.


Oyster River, British Columbia

Extended Response

15. Nabil made an error in simplifying the following expression.

a) Explain his mistake.

b) Determine the correct answer.

$$(12 \div 4)^4 + (5 + 3)^2$$
$$= (3)^4 + 5^2 + 3^2$$
$$= 81 + 25 + 9$$
$$= 106 + 9$$
$$= 115$$

16. A type of bacterium triples in number every 24 h. There are currently 300 bacteria.

a) Create a table to show the number of bacteria after each of the next seven days. Express each number of bacteria as the product of a coefficient and a power.

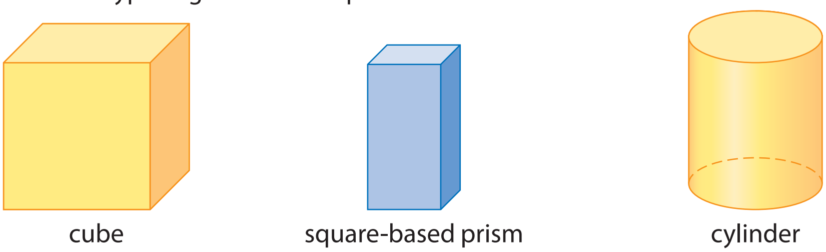
b) Determine a formula that will calculate the number of bacteria, B, after d days.

c) Use the formula to find the number of bacteria after 9 days.

d) How many were there 24 h ago? Explain your reasoning.

Math Link: Wrap It Up!

Create a mobile that uses at least three different types of regular three-dimensional shapes such as a cube, a square-based prism, and a cylinder. You may wish to choose a different type of geometric shape to build as well.



- Choose whole-number dimensions between 10 cm and 20 cm for each shape.
- Use a ruler and a piece of construction paper or other heavy paper to draw a net for each shape.
- Build each shape.
- Use expressions in exponential form to label the surface area and the volume of each shape.
- Evaluate each expression. Show all of your work.
- Make your mobile. Use colour and creativity!

Challenges

Develop Your Own Online Tournament

You are a game designer! You have developed an online computer game for you and your friends to play. Your goal is to make it available to a wider online audience one day. What kind of game is it?

Suppose the first tournament you hold has 16 players entered. You want to create a single-elimination draw to determine a winner.

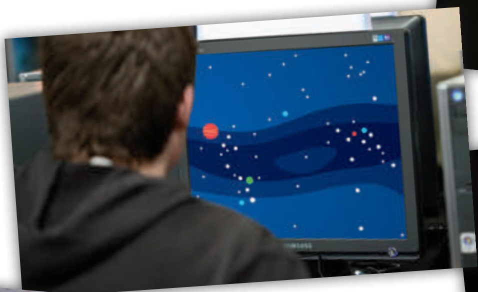
1. Make a draw for your competition. Show your work and explain your thinking.

2. What would be the next largest number of players that would fill a draw so that no player receives a bye?

3. a) Your dream is to hold a huge worldwide tournament. It will be a single-elimination draw involving close to one million players. What pattern from the first tournament can help you set up a tournament for a larger number of competitors?

b) What would be the ideal number of players closest to one million that would determine one winner fairly? Explain.

c) How many rounds would be played, based on the number of players in part b)?



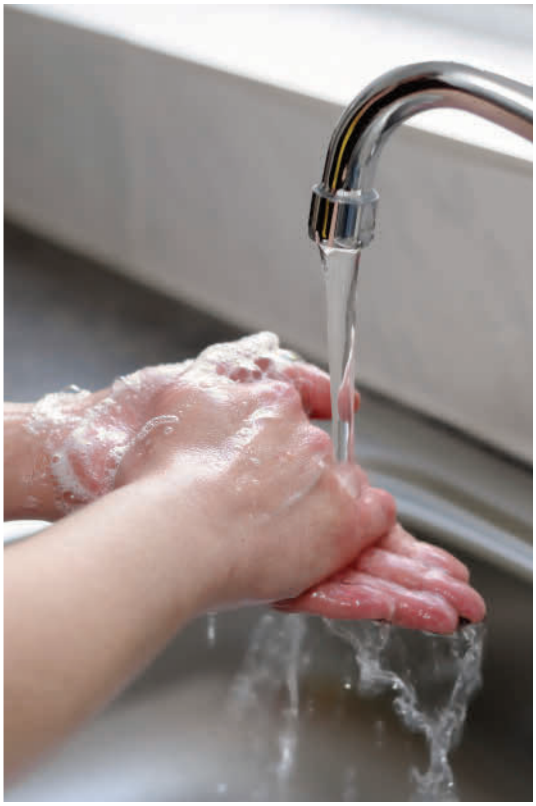
Literacy Link

A *draw* is a method to determine which players compete against each other in a tournament. In a single-elimination draw, competitors who lose a single match are knocked out of the tournament.

Literacy Link

A *bye* occurs when there is an odd number of teams or players in a tournament draw. When players are paired to compete against one another, there will be one player without an opponent. That player gets a bye, or pass, into the next round of the tournament.

Stopping the Spread of Harmful Bacteria



Health authorities encourage people to wash their hands, with good reason. If a single *Escherichia coli* bacterium finds its way under your skin, the number of bacteria can double every 20 min under ideal conditions. This means that one bacterium in a Petri dish could grow to about 500 000 bacteria in approximately 6 h and 20 min.

You are a public health official. You have been assigned the responsibility of creating a presentation about bacteria. Your presentation will inform the public about how quickly a population of bacteria can grow. It will also discuss the importance of reducing bacteria growth.

Research facts about a type of bacterium of your choice. Your presentation will include the following information:

- how long it would take for one bacterium to expand to a billion or more bacteria
- what mathematical terms you could use to describe this growth
- how you could determine the doubling rate of the bacterium you chose
- ways to reduce the growth of harmful bacteria

Use a format of your choice that would be effective for communicating your message.