CHAPTER

5

Introduction to Polynomials

Many people have an interest in tricks, games, and puzzles. Sometimes, it is difficult to determine how puzzles work. However, with a few simple techniques you can usually figure them out.

Illusions are one type of puzzle. In the illusion shown here, there are several faces. Examine the picture. How many faces can you find?

In this chapter, you will use polynomials, a part of algebra, to help explain how games, puzzles, and number tricks work.

What You Will Learn

- to demonstrate an understanding of polynomials
- to model, record, and explain addition and subtraction of polynomials

WWW Web Link

The optical illusion shown here is not the full image. To look for faces in the full image, go to www.mathlinks9.ca and follow the links. Try to find at least 8 faces.

Digital rights not available.

Key Words

algebra
term
polynomial
monomial
binomial
trinomial
degree of a term
degree of a polynomial
like terms

Digital rights not available.

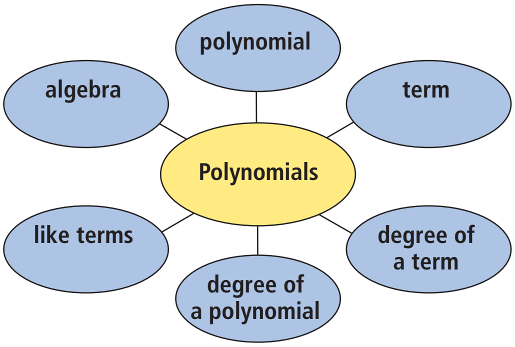
Literacy Link

A concept map can help you visually organize your understanding of math concepts.

Create a concept map in your math journal or notebook. Make each oval large enough to write in. Leave enough space to draw additional ovals. As you work through the chapter, complete the concept map.

For each term, write a definition and provide an example.

Discuss your ideas with a classmate. You may wish to add to or correct what you have written.


polynomial
algebra
term
Polynomials
like terms
degree of a term
degree of a polynomial

FOLDABLES™
Study Tool

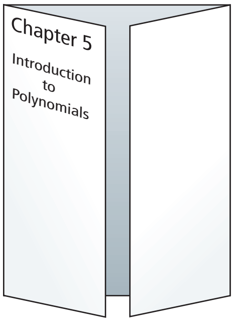
Making the Foldable

Materials

- sheet of 11 × 17 paper
- four sheets of 8.5 × 11 paper
- ruler
- scissors
- stapler

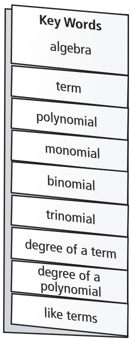
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Label it as shown.


Chapter 5
Introduction to Polynomials

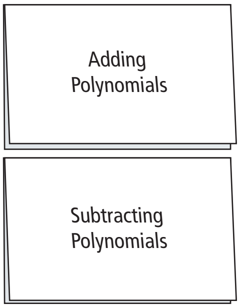
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. On one side, use a ruler to draw a line 4 cm from the top. Then, draw seven more lines at 3-cm intervals. Cut along the lines, forming nine tabs. Label the tabs as shown.


Key Words
algebra
term
polynomial
monomial
binomial
trinomial
degree of a term
degree of a polynomial
like terms

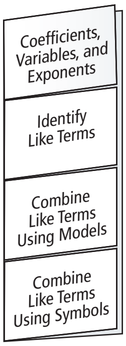
Step 3

Fold the long side of two sheets of 8.5 × 11 paper in half. Label them as shown.


Adding Polynomials
Subtracting Polynomials

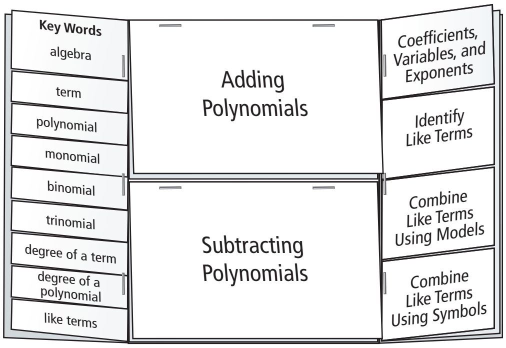
Step 4

Fold the long side of an 8.5 × 11 paper in half. Fold in four the opposite way. Make three cuts as shown through one thickness of paper, forming four tabs. Label the tabs as shown.


Coefficients, Variables, and Exponents
Identify Like Terms
Combine Like Terms Using Models
Combine Like Terms Using Symbols

Step 5

Staple the four booklets you made into the Foldable from Step 1 as shown.


Key Words
algebra
term
polynomial
monomial
binomial
trinomial
degree of a term
degree of a polynomial
like terms
Adding Polynomials
Subtracting Polynomials
Coefficients, Variables, and Exponents
Identify Like Terms
Combine Like Terms Using Models
Combine Like Terms Using Symbols

Using the Foldable

As you work through the chapter, write the definitions of the Key Words beneath the tabs on the left. Beneath the tabs in the centre panel and the tabs on the right, record notes and show worked examples.

On the front of the right flap of the Foldable, record ideas for the Wrap It Up! On the back of the centre panel, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

Math Link

Illusions, Puzzles, and Games

With an optical illusion, you are fooled into seeing something that is not really there. With a number trick, you are fooled into believing that a number has been magically conjured from thin air … unless you are able to figure out the trick!

Famous illusionist David Copperfield often uses a number trick as part of his program.

1. Try the number trick called Guess an Age several times using different ages. What do you find in Step 5? Why does it always work?

> **Guess an Age**
>
> **Step 1** Ask a person with a two-digit age to multiply the tens digit in their age by 5.
>
> **Step 2** Then, add 3.
>
> **Step 3** Then, double the sum from Step 2.
>
> **Step 4** Have the person add the value of the second digit of their age to the value in Step 3.
>
> **Step 5** Finally, subtract 6.

2. Try the number trick called Guess a Number.

> **Guess a Number**
>
> **Step 1** Have someone choose any whole number.
>
> **Step 2** Then, have that person roll a pair of dice and add the sum of the numbers from the top of the dice to the chosen whole number.
>
> **Step 3** Next, add the numbers on the bottom of the dice to the number.
>
> **Step 4** Have the person tell you the number from Step 3.

a) Explain how to find the person's original number.
b) How would you need to change the way you find the original number if you used three dice?

3. Find a number trick. Try it on your classmates. Explain why it works.

In this chapter, you will explore additional number tricks and games. You will use polynomials to explain how each one works. What number tricks do you know?

Digital rights not available.

WWW Web Link

To try other number puzzles, go to www.mathlinks9.ca and follow the links.

5.1

The Language of Mathematics

Focus on...

After this lesson, you will be able to...

- use mathematical terminology to describe polynomials
- create a model for a given polynomial expression

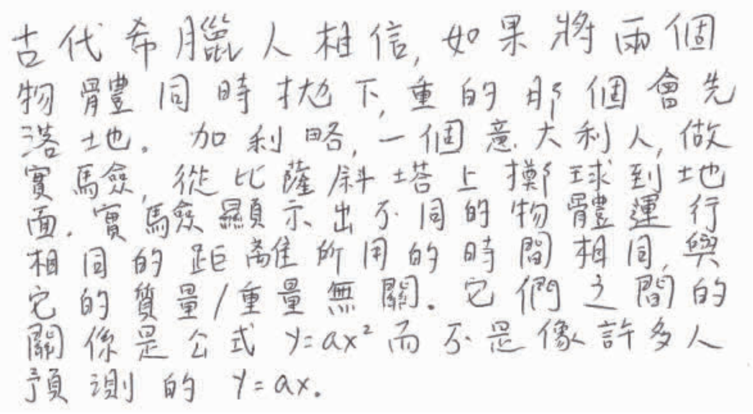
The Great Wall is the world's largest human-made structure. It stretches over 6700 km across China. The wall was created by joining several regional walls.

algebra

- a branch of mathematics that uses symbols to represent unknown numbers or quantities

Similarly, mathematics is a developing science made up of several branches, including arithmetic, geometry, and **algebra**. It is a science that studies quantity, shape, and arrangement. As with any science, mathematics comes with its own unique language. The language of mathematics is universal. It can be understood anywhere in the world.

Look at the following paragraph. How much of it can you read? What languages do you think the paragraph contains?



古代希臘人相信，如果將兩個
物體同時拋下，重的那個會先
落地。加利略，一個意大利人，做
實驗，從比薩斜塔上擲球到地
面。實驗顯示出不同的物體運行
相同的距離所用的時間相同，與
它的質量/重量無關。它們之間的
關係是公式 $y=ax^2$ 而不是像許多人
預測的 $y=ax$。

Would the algebraic equations be any different if the paragraph was written in any other modern language?

Explore the Language of Algebra

1. a) For the algebraic expression $5a + 4b$, what terminology can you use to describe the numbers 4 and 5, and the letters a and b?

b) What terminology can you use to describe the expression $-7x^2$ and its parts?

2. Make up a real-life situation and write an algebraic expression for it. What do the parts of your expression represent?

3. Algebraic expressions can have different numbers of **terms**.

Number of Terms	Examples
1	5 $7x$ $-3ab$ $\frac{y}{2}$
2	$5 + x$ $3x^2 - 2$ $7xy + z^2$
3	$1 - x + y$ $2x^2 - 3x + 5$ $a + b + c$

Write other examples of expressions with one term, two terms, and three terms.

term

- an expression formed from the product of numbers and/or variables
- $9x$ is a term representing the product of 9 (coefficient) and x (variable)
- a constant value, such as 5, is also a term

4. A study of algebra includes working with **polynomials**. They are named by their number of terms. How do the names *monomial*, *binomial*, and *trinomial* relate to the number of terms in an expression?

What common words do you know that have prefixes of *mono*, *bi*, and *tri*?

polynomial

- an algebraic expression made up of terms connected by the operations of addition or subtraction
- $3x^2 - 4$ has two terms. $3x^2$ and 4 are connected by the operation of subtraction.

Reflect and Check

5. Look at the algebraic equations in the paragraph written in Chinese on the previous page. Use as much algebraic terminology as you can to describe them.

Language Link

The word *algebra* comes from Arabic. The word originated in Iraq. Around A.D. 830, Mohammad al-Khwarizmi of Baghdad wrote a book called *Hisab al-jabr w'al-muqabalah*. This book summarized Hindu understandings of equations and how to solve them. The whole title was too hard for some Europeans so they kept only the word *al-jabr*. We get the term *algebra* from that Arabic word.

WWW Web Link

For more information about the history of algebra, go to www.mathlinks9.ca and follow the links.

Literacy Link

Types of Polynomials

Some polynomials have specific names.
Monomial: one term
Binomial: two terms
Trinomial: three terms

All expressions with one or more terms are called *polynomials*. *Polynomial* means many terms.

Link the Ideas

Example 1: Name Polynomials by the Number of Terms

For each expression, identify the number of terms and whether it is a monomial, binomial, trinomial, or polynomial.

a) $4xy + 3$
b) $7a^2 - 2ab + b^2$
c) $5x^2 + y^2 + z^2 - x - 6$
d) 13

Solution

Expression	Number of Terms	Name
a) $4xy + 3$	2	binomial
b) $7a^2 - 2ab + b^2$	3	trinomial
c) $5x^2 + y^2 + z^2 - x - 6$	5	polynomial
d) 13	1	monomial

What are the two terms in $4xy + 3$?

Show You Know

For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a) $5j^2$
b) $3 - m^2$
c) $ab^2 - ab + 1$
d) $-4x^2 + xy - y^2 + 10$

degree of a term

- the sum of the exponents on the variables in a single term (monomial)
- $3xz$, or $3x^1z^1$, has degree 2, since $1 + 1 = 2$
- $5x^2y$ and $-2b^3$ are terms of degree 3

degree of a polynomial

- the degree of the highest-degree term in a polynomial
- In $7a^2 - 3a$, the degree of the first term is 2, and the degree of the second term is 1. The highest degree is 2, so the degree of the polynomial is 2.

Example 2: Identify the Number of Terms and Degree of a Polynomial

What is the number of terms and the degree of each polynomial?

a) $4x^2 + 3$
b) $7a^2 - 2ab + b^2$
c) $5x + z - 6$
d) 7

Solution

a) The polynomial $4x^2 + 3$ has two terms.
The term with the highest degree is $4x^2$.
Its degree is 2.
The degree of the polynomial is 2.

What degree does the constant term 3 have?

b) The polynomial $7a^2 - 2ab + b^2$ has three terms.
Each of the three terms has a degree of 2.
The degree of the polynomial is 2.

When a variable has no exponent, the value of the exponent is 1. How do you know $2ab$ has a degree of 2?

c) The polynomial $5x + z - 6$ has three terms.
Both of the terms $5x$ and z have a degree of 1.
The degree of the polynomial is 1.

d) The number 7 is one term that is constant.
A constant term has a degree of 0.
The degree of the polynomial is 0.

Show You Know

What is the number of terms and the degree of each polynomial? Explain each answer.

a) $1 - 3x$ **b)** $4x - 3xy + 7$

c) $-27b^2$ **d)** 99

Literacy Link

When a term has more than one variable, the variables are usually written in alphabetical order.

Examples: $5ab$, $-12x^2y^2$

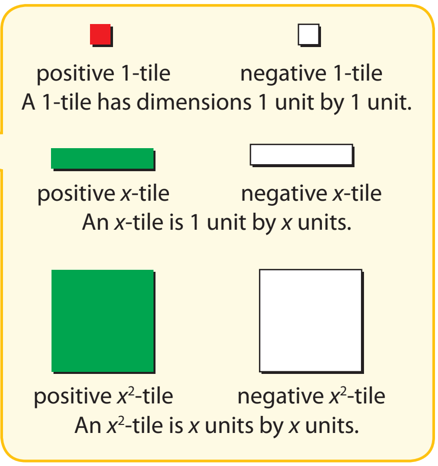
Example 3: Model Polynomials

Model each polynomial.

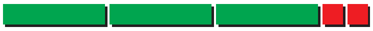
a) $3x + 2$ **b)** $-x^2 - 2$ **c)** $2x^2 + 3 - x$

Solution

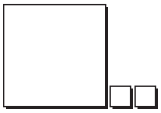
You can model each polynomial using algebra tiles.


positive 1-tile
negative 1-tile
A 1-tile has dimensions 1 unit by 1 unit.
positive x-tile
negative x-tile
An x-tile is 1 unit by x units.
positive x^2-tile
negative x^2-tile
An x^2-tile is x units by x units.

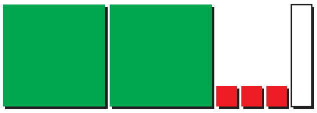
a) $3x + 2$



b) $-x^2 - 2$



c) $2x^2 + 3 - x$

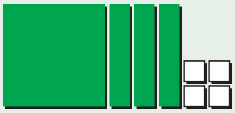


Tiles can be either horizontal or vertical.

Show You Know

a) Model $-x^2 + 4x - 3$.

b) What expression is shown by the algebra tile model?



Web Link

For practise using virtual algebra tiles, go to www.mathlinks9.ca and follow the links.

Key Ideas

- Algebra is a branch of mathematics that uses symbols to represent unknown numbers or quantities. The symbols are often letters and are called variables.
- Polynomials are made up of terms. Terms are connected by addition or subtraction.

  $3x^2 + 2x - 7$ has three terms.
- Polynomials can have one or more terms. Some polynomials have specific names.

Name	Number of Terms	Example
monomial	1	$6x^2$
binomial	2	$3a^2 - 5$
trinomial	3	$-w^2 + 5w + 1$
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$

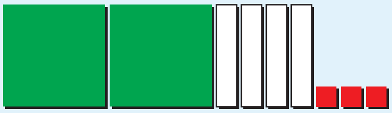
- Each algebraic term has a degree. You can find the degree of a term by adding the exponents of the variable(s) in the term.

  $3x$ has degree 1. $-5x^2y$ has degree 3.
- A polynomial has the same degree as its highest-degree term.

  $x^2 + 5x - 7$ has degree 2.

  12 has degree 0.
- You can use models, such as algebra tiles and diagrams, to represent some polynomials.

  $2x^2 - 4x + 3$

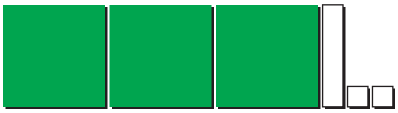


Check Your Understanding

Communicate the Ideas

1. Identify at least three mathematical words or phrases you could use to refer to the polynomial $-5 + x^2$.

2. Sonja and Myron are discussing the algebra tile model shown below.



Sonja says, "The model represents the expression $3x^2 + x + 2$." Myron claims, "It represents $3x^2 - x - 2$." Who is correct? How do you know?

3. Give two examples of a polynomial that satisfies all statements.
- consists of two terms
- contains two variables
- has degree 2
- one term is of degree 1 and has a coefficient of 1

4. When is it acceptable not to write the 1 in an algebraic expression? When must you write the 1? Give examples.

Practise

For help with #5 to #7, refer to Example 1 on page 176.

5. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a) $3x^2 - 5x - 7$
b) $-11a$
c) $c^2 + cf + df - f^2$
d) 8

6. What is the number of terms and what is a name for each expression?

a) n
b) $6 + 4x - x^2$
c) 0
d) $p^2 + 3pq$

7. Refer to the polynomials below to answer each question.

$6x$	-15
$3x - y$	$4c^2 - cd$
$7 + a + b$	$3m^2 - 4mn - 9n^2 + 1$

a) Which ones are monomials?
b) Which ones are trinomials?
c) Which ones have two terms?

For help with #8 to #10, refer to Example 2 on pages 176–177.

8. For each polynomial, what is the degree and number of terms?

a) $4 - b$
b) $fg + 2g$
c) $8x^2 - xy - y^2$

9. State the degree and number of terms for each polynomial.

a) $3xy + 1$
b) $11k^2 + 7k - 5$
c) 6

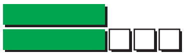
10. Refer to the polynomials below to answer each question.

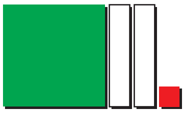
$3b^2$	$2 + p$
$4st + t - 1$	$2x^2 - y^2$

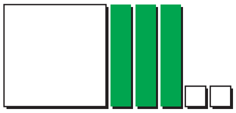
a) Which ones are binomials?
b) Which ones have degree 2?
c) What is the variable in the monomial?
d) Which polynomials have a constant term?

For help with #11 to #14, refer to Example 3 on page 177.

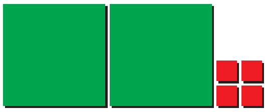
11. What expression is represented by each set of algebra tiles?

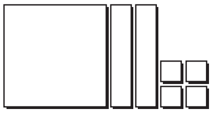
a)


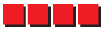
b)


c)


12. Write the expression represented by each set of algebra tiles.

a)


b)


c)


13. Model each polynomial.

- **a)** $x^2 + x - 1$
- **b)** $3x + 2$
- **c)** $-2x$

14. Use a model to represent each polynomial.

- **a)** $-x^2 + 3$
- **b)** $2x^2 - 3x$
- **c)** 8

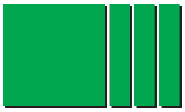
Apply

15. Represent each of the following with a diagram and an expression.

- **a)** binomial
- **b)** monomial of degree 1
- **c)** monomial of degree 2 with a coefficient of 9
- **d)** polynomial with four terms that is of degree 2

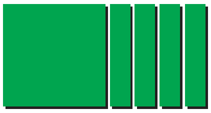
16. Use your knowledge of algebra tiles to answer the following questions.

- **a)** How are the dimensions of a 1-tile and an x-tile related?
- **b)** The rectangle shown was formed using an x^2-tile and three x-tiles. What is an expression for the length of the rectangle?



17. Write an algebraic expression for each of the following.

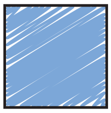
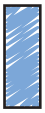
- **a)** the product of 6 and x
- **b)** the sum of $2x$ and 3
- **c)** the length of the rectangle below, which is made from algebra tiles



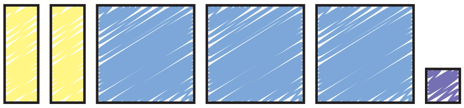
18. Make a model of an algebraic expression that includes at least one x^2-tile, at least two x-tiles, and two 1-tiles. Use materials or a diagram. Then, use symbols to show your expression. What type of polynomial is it?

19. For the polynomial $6x^2 - 5$, state the following.

- **a)** number of terms
- **b)** coefficient of the first term
- **c)** number of variables
- **d)** degree of polynomial
- **e)** constant term

20. Let  represent x^2,  represent x, and represent 1. The same diagrams in yellow represent negative quantities.

- **a)** What is an expression for the polynomial shown?



- **b)** Make up a trinomial. Draw diagrams to represent your trinomial.

21. Write each statement as an algebraic expression. Include what your variables represent.

- **a)** Eight and a number are added together.
- **b)** Omar has some money in his wallet. How much money does he have after a friend gives him \$5?
- **c)** A page is 4 cm longer than its width.
- **d)** The product of a number and 5 is increased by 2.
- **e)** The result of 3 times the number of people decreased by 21.

22. Describe a situation that could be modelled by each given polynomial.

- **a)** $3x + 5$
- **b)** $10 - x$

23. Marion gives French lessons in the evening. She charges \$20 for adults and \$15 for children. The expression $20a + 15c$ represents her earnings.

a) What do the variables a and c represent?

b) How much does Marion make if she gives lessons to four adults and nine children? Show your work.

c) Write a new expression for Marion's earnings if she charges \$3 more for adults and \$2 more for children.

24. Tickets for a school concert are \$10 for adults and \$5 for students. Write an expression that shows the total income for the school concert. Tell what your variables represent.

25. A hockey league awards teams two points for a win, one point for a shoot-out loss, and no points for a loss in regulation time.

a) Write an algebraic expression to represent the total points for a hockey team.

b) What variable(s) did you use? Indicate what each variable represents.

c) In the first 20 games of the season, Team A had 12 wins and 4 shoot-out losses. How many losses in regulation time did the team have?

d) What were the total points for Team A?

e) Team A was tied with Team B after 20 games. However, Team B had a different record than Team A. Show two possible records for Team B. Use your expression to show that the two hockey teams had the same number of total points.

26. A banquet hall can be rented for parties. An expression for the rental cost is $5n + 75$, where n is the number of people.

a) What type of polynomial is $5n + 75$, and what is its degree?

b) What could the numbers 5 and 75 represent?

c) How much does it cost to rent the banquet hall for 150 people?

Extend

27. On a true/false test, there is a penalty for incorrect answers. Miranda's teacher advises the students not to guess at any of the 25 questions. The teacher awards 2 points for a correct answer, -1 point for a wrong answer, and 0 points if the question is not answered.

a) Write a polynomial to represent a student's score on this test.

b) What are the maximum and minimum scores possible on this test? Explain.

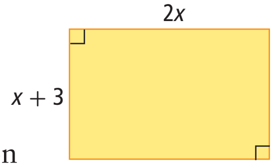
c) What are all of the possible scores if Miranda got 20 questions correct? Explain.

28. What is the degree of

$xy - abx + cdy - qr - prqz$

if x, y, and z are variables and a, b, c, d, p, q, and r are coefficients?

29. Ricardo draws the following rectangle with dimensions in metres.



a) What is an expression for the perimeter of the rectangle?

b) Write an equation showing how the length and width of the diagram would be related if the dimensions given were for a square.

c) Solve your equation in part b) to find the value of x. Show your work.

30. Create a polynomial satisfying the following conditions:

- contains three variables
- has three terms
- is of degree 2
- has a constant term, 3

31. Deidra is training for a triathlon. From her training, she knows that she can swim at 1.3 km/h, cycle at 28 km/h, and run at 12 km/h.

a) Write the formula
distance (d) = speed × time
using variables of your choice for speed and time. Tell what each variable represents.

The distance for each leg of the competition is different. In the table, this difference is shown using subscripts.

b) Use your formula to complete the table.

Part of Race	Distance (km)	Speed (km/h)	Time (h)
swim	d_1	1.3	$\frac{d_1}{1.3}$
cycle	d_2	28.0	
run	d_3	12.0	

c) Write a trinomial to model Deidra's total time for a triathlon.

d) A triathlon includes a 1.5-km swim, a 40-km cycle, and a 10-km run. How long will it take Deidra to complete this triathlon? What assumptions are you making?

e) If Deidra could maintain the same speeds, how long would it take her to complete a triathlon that is a 3.8-km swim, 180-km cycle, and 42.2-km run?

WWW Web Link

The Ironman Canada Triathlon in Penticton, B.C., involves a 3.8-km swim, 180-km cycle, and 42.2-km run. The times of recent winners are impressive. Top men's times are around 8 h, while women's times are around 9 h. For information about the Ironman Canada Triathlon and about the history of the Ironman competition, go to www.mathlinks9.ca and follow the links.

Math Link

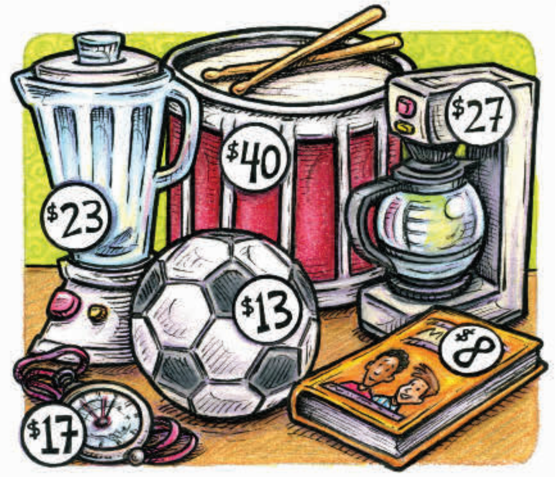
You want to be a contestant on a game show. In order to get on the show, you must show how to spend exactly $100 by choosing from the items shown.

You may purchase some or all of the six items, and as many of a single item as necessary.

a) Find at least six answers that would get you on the game show.

b) Write an algebraic expression for one of your combinations in part a). What is an equation for this same combination?

c) Is it possible to spend $100 choosing all different items? Explain.



5.2

Equivalent Expressions

Focus on…

After this lesson, you will be able to…

- use algebra tiles and diagrams to show whether expressions are equivalent
- identify equivalent expressions that are polynomials
- combine like terms in algebraic expressions

Materials

- concrete materials, such as algebra tiles

Today's space program requires extensive use of algebra. Computer programs control shuttle flights and manipulate the Canadarm. They also control conditions inside the International Space Station. These programs use algebraic models, expressions, and equations. Where else in the real world is algebra used?

Explore Combining Like Terms

The astronauts on the space shuttle have a limited amount of living space. They eat, sleep, and relax in a rectangular space with a width of only about 3.2 m and a length that is 0.8 m greater than the width.

1. What is the length of the living space? How do you know?

2. Draw and label a diagram of the rectangular living space. Find the perimeter of the rectangle. How did you find the perimeter?

3. Draw another rectangle. The length is still 0.8 m greater than the width but you have no known value for the width. What could you use to represent the width of the rectangle? What would be an expression for the length of this rectangle?

4. Write an expression for the perimeter of the second rectangle.

5. How many terms are in your expression for the perimeter?

6. Use materials or a diagram to model your expression for the perimeter.

7. Rearrange your model so similar objects, shapes, or variables are all together. Combine the similar objects, shapes, or variables. What is an equivalent expression for the perimeter?

Reflect and Check

8. What do you think *like terms* means? Give examples to support your ideas.

9. How do you combine like terms in polynomials? Explain with examples.

Link the Ideas

Example 1: Identify Coefficients, Variables, and Exponents

For each expression, identify the coefficient, the variable(s), and the exponent of each variable.

a) $3w$ **b)** a^2 **c)** $-4xy$ **d)** $-g$

Solution

Expression	Coefficient	Variable(s)	Exponents of the Variable(s)
a) $3w$	3	w	1
b) a^2	1	a	2
c) $-4xy$	−4	x and y	1 and 1
d) $-g$	−1	g	1

Literacy Link

Even if a term has two variables, it always has only one coefficient. Example: A term would be written as $-6xy$, *not* as $-2x3y$.

Show You Know

Give the coefficient, the variable(s), and the exponent of each variable.

a) $3c^2$ **b)** $-x$ **c)** b **d)** $7st^2$

Example 2: Identify Like Terms

Identify the like terms in each group.

a) $5b^2$ $3cb$ $-2b$ $7c$ $6b$

b) $3x^2$ $4xy$ $-2x^2$ $7x^2$ $\frac{1}{2}y$

c) $3pq$ 11 $-4q^2$ -3 pq

like terms

- terms that differ only by their numerical coefficients
- examples of like terms are
  - $3x$ and $-2x$
  - $6y^2$ and $-4y^2$
  - $-5xy$ and yx
  - 17 and −8

Solution

a) $-2b$ and $6b$ are like terms. Both have a variable b with an exponent of 1. All the other terms are unlike.

b) $3x^2$, $-2x^2$, and $7x^2$ are like terms. Each of them has a variable x with an exponent of 2. The other terms are unlike.

c) $3pq$ and pq are like terms. Both have variables p and q, each with an exponent of 1. The terms 11 and −3 are also like terms.

Show You Know

a) Give an example of three like terms.

b) Identify the like terms in the following group: $6t$ $3s$ $6t^2$ $6st$ $-8s$

Example 3: Combine Like Terms

Combine like terms in each expression.

a) $4x - 2x + 3 - 6$
b) $2x^2 + 3x - 1 + x^2 - 4x - 2$
c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

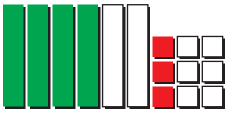
Solution

Method 1: Use a Model

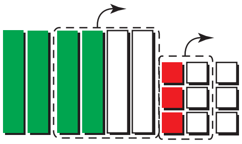
Strategies

Model It

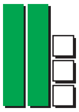
a) $4x - 2x + 3 - 6$
You can use algebra tiles to represent each term.



Group the tiles to form zero pairs and remove the pairs.



Write an expression for the remaining tiles.
$2x - 3$

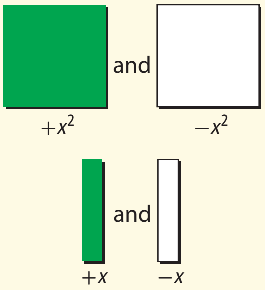


So, $4x - 2x + 3 - 6 = 2x - 3$.

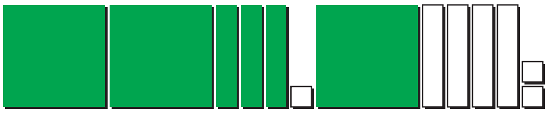
Like terms can be combined to simplify expressions.

🟥 ($+1$) and ☐ (-1) have a combined value of zero and are called a *zero pair*.

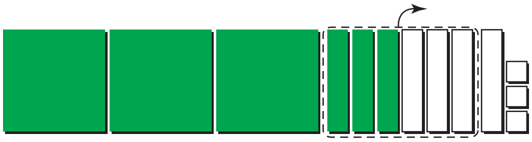
The same is true for



b) $2x^2 + 3x - 1 + x^2 - 4x - 2$



Group like terms and remove the zero pairs.



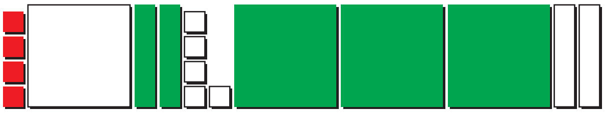
$$\begin{aligned} &2x^2 + 3x - 1 + x^2 - 4x - 2 \\ =\ &3x^2 - 1x - 3 \\ =\ &3x^2 - x - 3 \end{aligned}$$

Literacy Link

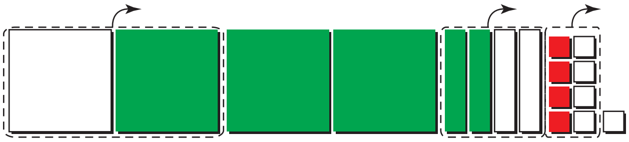
Any term with a variable having a coefficient of 1 can be written without its numerical coefficient. However, the sign must remain. Example:
$-1x = -x$
$+1x = x$
$-1x^2 = -x^2$
$+1x^2 = x^2$

Coefficients without a sign are positive.

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$



Group like terms and remove the zero pairs.



$4 - x^2 + 2x - 5 + 3x^2 - 2x = 2x^2 - 1$

Method 2: Use Symbols

a) Add or subtract the coefficients of like terms.

$4x - 2x + 3 - 6 = 2x - 3$

b) A polynomial with more than one term can be written in different orders. Rearrange by grouping like terms.

$$\begin{aligned} &2x^2 + 3x - 1 + x^2 - 4x - 2 \\ &= 2x^2 + x^2 + 3x - 4x - 1 - 2 \\ &= 3x^2 - x - 3 \end{aligned}$$

c)
$$\begin{aligned} &4 - x^2 + 2x - 5 + 3x^2 - 2x \\ &= -x^2 + 3x^2 + 2x - 2x + 4 - 5 \\ &= 2x^2 - 1 \end{aligned}$$

Literacy Link

In algebra, terms are often arranged in descending order by degree. For example, $-3y + 4y^2 - 1$ is written as $+4y^2 - 3y - 1$ or $4y^2 - 3y - 1$. This makes it easier to compare expressions. Answers are usually written this way.

WWW Web Link

To learn more about the parts of a polynomial and combining like terms, go to www.mathlinks9.ca and follow the links.

Show You Know

Combine like terms.

a) $5x - 3x^2 + 2x - x^2$

b) $2x - 6 - 2x + 1$

c) $k - 2k^2 + 3 + 5k^2 - 3k - 4$

Key Ideas

- An algebraic expression is made up of terms. Each term can have any number of variables. Each variable has an exponent. A constant term, such as 9, has no variable.

Term	Coefficient	Variable(s)	Variable's Exponent
$6p^2$	6	p	2
$-x^2y$	-1	x, y	2 for x, 1 for y

- Like terms differ only by their numerical coefficients.

Like terms can be combined.	Unlike terms cannot be combined.
Like terms:	Unlike terms:
▪ $-7x$ and $3x$	▪ $6x$ and $3x^2$
▪ w^2, $3w^2$, and $0.5w^2$	▪ m^2n and $4mn^2$
▪ 6 and 15	▪ 7 and $7d$

Check Your Understanding

Communicate the Ideas

1. Using models, show how you know that $s - 5s$ combines to give $-4s$.

2. Jean claimed that $3m^2 + 4m$ could be combined to give $7m^3$. Do you agree? Explain with diagrams.

3. Most people would agree that 2 cats + 5 cats = 7 cats and 7 cats − 2 cats = 5 cats. Use this information to support an argument for combining like terms and for being unable to combine unlike terms in algebra. Use examples with two different animals and two different variables.

4. Does a number in front of a variable affect whether or not you have like terms? Explain using examples.

Practise

For help with #5 to #7, refer to Example 1 on page 184.

5. What is the value of the coefficient and the number of variables for each term?

a) $-3z^2$ **b)** k **c)** 43

6. Determine the value of the coefficient and the number of variables for each term.

a) $4d$ **b)** $-prt$ **c)** $-8fg^2$

7. Use the following monomial expressions to answer the questions below:

$3x$	$4t$	x^2
$-ts$	xt	$2t^2$

a) Which have a coefficient of 1?

b) Which have two variables?

c) Which have only one variable with an exponent of 1?

d) Which have a coefficient of −1?

For help with #8 and #9, refer to Example 2 on page 184.

8. Identify the like terms in each group.

a) $2a \quad 5 \quad -7.1a \quad 9b \quad -c$

b) $3m \quad -2ab \quad \frac{4}{3}m \quad -2ad \quad m^2$

c) $-1.9 \quad 6p^2 \quad 5 \quad -2p \quad p^2$

9. Which terms are like terms in each group?

a) $-2k \quad 9 \quad 104k \quad 104f \quad -f^2$

b) $\frac{1}{2}ab \quad 0.5a \quad -4b \quad ab^2 \quad ab$

c) $-5 \quad 13d^2 \quad 5 \quad -10d \quad d^2$

For help with #10 to #12, refer to Example 3 on page 185.

10. Collect like terms.

a) $3x - 2x^2 + x - 2x^2$

b) $-4 - 2n^2 - 3n + 3 + 2n^2$

c) $2q - 4q^2 - 2 + 3q^2 + 2 - 3q$

d) $-4c + 3 + 5c - 7$

e) $h^2 - 3h + 4h^2 + 2h$

f) $3j - 5 + 2j^2 - 1 + 2j - 3j^2$

11. Simplify by collecting like terms.

a) $2d - 3d^2 + d^2 - 5d$

b) $y^2 + 2y - 2y^2 + y$

c) $-p + 4p^2 + 3 - 3p^2 - 5 + 2p$

d) $m - 4 + 6 + 3m$

e) $q^2 - 3q + 2q^2 - q$

f) $5w - 3 + w^2 - 2w - 4w^2 - 1$

12. Which expressions are equivalent to the simplified expression $-3x^2 + x - 4$?

A $-4 + 3x^2 + x$

B $x - 4 -3x^2$

C $x^2 + 2 - 4x^2 + 3x - 6 - 2x$

D $-3 - 5x^2 + x + 1 + 2x^2$

E $2x - 2 + x^2 - x - 4x^2 - 2$

F $-4 - 3x - 3x^2 - 0 + 5x^2 + 4x - 6x^2$

Apply

13. Jessica and Taz are working on a measurement problem. Their calculations involve combining Jessica's measurement of 2 m and 32 cm with Taz's measurement of 1 m and 63 cm. Jessica claims you find the answer just like in algebra. Do you agree? Explain.

14. Describe a real-life situation that could be represented by each expression.

a) $m - 3$ **b)** $2p + 5$

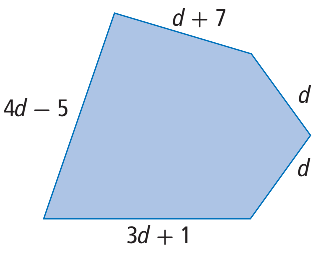
15. For each of the following polynomials, write an equivalent expression with six terms.

a) $2p^2 - 3p + 2$ **b)** $-3x^2 + 5x - 4$

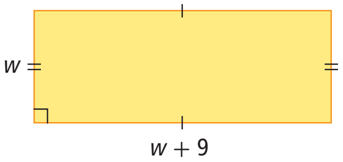
c) $4r^2 - 2q^2 - 3qr$

16. Write an expression for the perimeter of each figure. Then, combine like terms if possible.

a)



b)



17. a) Draw a figure with a perimeter that is represented by $(5s) + (3s - 2) + (s + 6)$, where each value in parentheses represents the length of one side.

b) Simplify the expression for the perimeter.

18. A student council decides to raise money by organizing a dance. The cost of a band is \$700. The student council decides to sell tickets at \$5 each.

a) Write an expression for the profit that the student council would make. What does your variable represent?

b) If 250 students pay to attend the dance, what is the profit?

c) Estimate, then calculate, the minimum number of students who will need to buy tickets for the student council to make a profit.

19. A heating company charges \$60 per hour plus \$54 for a service call. Let n be the number of hours the technician works at your house.

a) What expression represents the total cost of the job?

b) What is the cost for a job requiring 2 h?

c) The company charges half as much for a second technician. Write an expression showing the total cost if two technicians install a new furnace. Simplify your expression by combining like terms.

20. A publisher makes books for a number of distributors. For one book, the charge to the distributor is represented by a fixed cost of \$3000 plus \$16 per book.

a) Write an expression for the cost that a distributor is charged for b copies of this book.

b) How much do 600 books cost?

c) What is the cost per book if 600 are ordered?

d) What is the cost per book if 1000 are ordered?

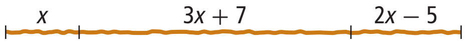
21. Raj was told to write an expression equivalent to $3x - 8 - 5x + 9$.

$3x - 8 - 5x + 9$
$= 3x - 5x - 8 + 9$
$= 2x - 1$

a) What errors did he make?

b) Show the correct response.

22. The diagram represents a piece of string.



a) What is an expression for the total length of the string?

b) Combine like terms to get the simplest expression possible for the length of the string.

Extend

23. When would the expressions $x + y + 3$ and $x + w + 3$ be equal? How do you know?

24. A department store marks up wholesale prices 40% to get its retail or selling price.

a) Complete the following table. The first row has been done for you.

Wholesale Price ($)	Expression For Retail Price	Retail Price ($)
8.00	$8 + (0.4)(8)$	11.20
12.00		
30.00		
x		

b) How could you find the retail price if the wholesale price is $x + 10$ dollars? Show two ways to find the answer.

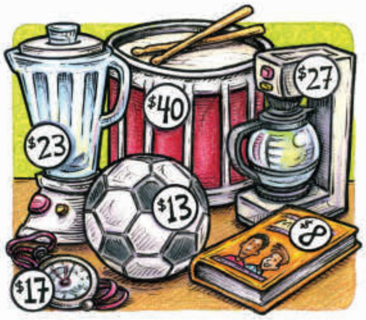
25. Zip Publishers will print posters for fundraising events for an initial cost of $100 plus $2 per poster. Henry's Printers charges $150 plus $1 per poster.

a) Write an expression for each company, showing the total cost for any number of posters.

b) What is the cost of 125 posters from each company?

c) What is the total cost if you print 200 posters at each company? Show two different ways to find the answer.

Math Link



Refer to the Math Link for section 5.1 on page 182. Represent each item with a variable:

a = blender
b = watch
c = book
d = soccer ball
e = drum
f = coffeemaker

a) Rewrite all your combinations that add to $100, using the letters a to f. Arrange each combination in alphabetical order. For example, $a + e + d + 3c$ would be written as $a + 3c + d + e$.

b) The example in part a) can be used to find other combinations. Notice that e has a value of $40. What other items from the list have a value of $40? By substituting into e the letter or letters that combine to a total value of $40, you arrive at another answer. Do not forget to combine like terms and arrange each expression in alphabetical order. What other combinations can you find using substitution?

c) If you were asked to find combinations of the items that add to $101, how could you use algebra to help you? Give two ways that algebra could help you.

5.3 Adding and Subtracting Polynomials

Focus on…

After this lesson, you will be able to…

- add polynomial expressions
- subtract polynomial expressions
- solve problems using the addition and subtraction of polynomials

A music store rents out a drum kit for $55 per month, plus a deposit of $30. Is there a pattern? How could you use a polynomial expression to represent this pattern?

Explore Adding or Subtracting Polynomial Expressions

1. In first semester, Kira decides to play drums for music class. To rent a drum kit, it costs $55 per month, plus a $30 deposit.

a) What is the total cost of renting the drum kit for three months, including the deposit?

b) Write an expression to show the total cost for any number of months, including the deposit. Tell what your variable represents.

2. In second semester, Kira decides to play electric guitar. To rent an electric guitar, it costs $22 per month, plus a $20 deposit.

a) What is the total cost of renting an electric guitar for three months, including the deposit?

b) Write an expression to show the total cost for any number of months, including the deposit.

Use the same variable that you used in #1. Why might you be able to use the same variable?

3. Mark wants to learn to play both drums and electric guitar. What is an expression for the total cost of renting a drum kit and a guitar for any number of months, including deposits? Then, show how to find a simpler expression.

4. What is an expression for the difference between the cost of renting a drum kit and the cost of renting a guitar for any number of months, including deposits? Then, show how to find a simpler expression.

Reflect and Check

5. **a)** Describe how to add or subtract polynomial expressions.

b) Why was it necessary to use the same variable for each expression?

6. Make up your own situation that involves the rental of two items. Write an expression for the total cost and for the difference in cost.

Link the Ideas

Example 1: Add Polynomials

Add $3x - 4$ and $2x + 5$. Simplify your answer by combining like terms.

What is another way to model the polynomials?

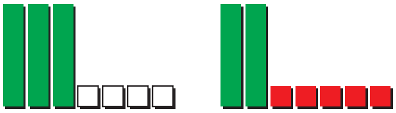
Strategies

Model It

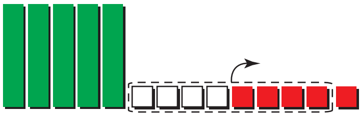
Solution

Method 1: Use a Model

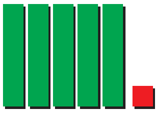
You can use algebra tiles to model each polynomial.



Arrange the model so that like objects are together. Remove zero pairs if necessary.



Rewrite the model in simplest form.



$(3x - 4) + (2x + 5) = 5x + 1$

Method 2: Use Symbols

You can combine like terms.

$$\begin{aligned} & (3x - 4) + (2x + 5) \\ &= 3x + 2x - 4 + 5 \\ &= 5x + 1 \end{aligned}$$

Show You Know

Use two methods to show each addition of polynomials. Give your answers in simplest form.

a) $(2a - 1) + (6 - 4a)$

b) $(3t^2 - 5t) + (t^2 + 2t + 1)$

Digital rights not available.

Example 2: Determine Opposite Expressions

What is the opposite for each of the following?

a) $3x$ **b)** -2

c) $4x - 1$ **d)** $a^2 - 3a + 2$

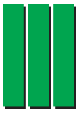
Strategies

Model It

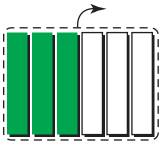
Solution

How might you use a diagram to model $3x$?

a) You can use algebra tiles to represent $3x$.



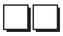
Add three negative x-tiles to give zero.



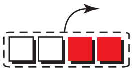
The opposite of $3x$ is $-3x$.

You can also use inspection.
$3x$ is positive. The opposite of positive is negative.
The opposite of $3x$ is $-3x$.

b) You can use algebra tiles to represent -2.



Add two positive 1-tiles to give zero.



The opposite of -2 is $+2$ or 2.

You can also use inspection.
-2 is negative. The opposite of negative is positive.
The opposite of -2 is $+2$ or 2.

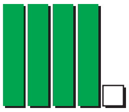
Art Link

Opposites can be used to create interesting optical illusions.

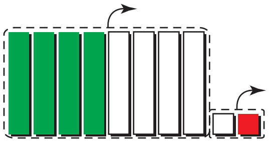
Look at the dot in the center of the rings. Lean forward and backward. The rings appear to rotate in opposite directions.

Digital rights not available.

c) You can use algebra tiles to represent $4x - 1$.



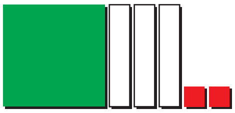
Add four negative x-tiles and one positive 1-tile to give zero.



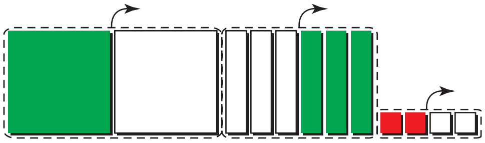
The opposite of $4x - 1$ is $-4x + 1$.

You can also use inspection.
$4x$ is positive. The opposite of positive is negative.
The opposite of $4x$ is $-4x$.
1 is being subtracted. The opposite of subtracting 1 is adding 1.
The opposite of $4x - 1$ is $-4x + 1$.

d) You can use algebra tiles to represent $a^2 - 3a + 2$.



Add a negative a^2-tile, three positive a-tiles, and two negative 1-tiles.



The opposite of $a^2 - 3a + 2$ is $-a^2 + 3a - 2$.

You can also use inspection.
a^2 is positive. The opposite of positive is negative.
The opposite of a^2 is $-a^2$.
$3a$ is being subtracted. The opposite of subtracting $3a$ is adding $3a$.
2 is being added. The opposite of adding 2 is subtracting 2.
The opposite of $a^2 - 3a + 2$ is $-a^2 + 3a - 2$.

Show You Know

What is the opposite of each expression? Justify your answer.

a) x

b) $5 - 3x$

c) $7x^2 + 5x - 1$

Example 3: Subtract Polynomials

Subtract $2x + 3$ from $3x - 4$. Simplify your answer by combining like terms.

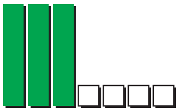
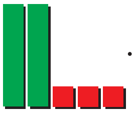
Solution

$(3x - 4) - (2x + 3)$

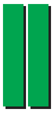
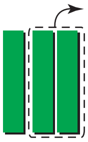
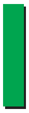
Strategies

Model It

Method 1: Use a Model
You can use algebra tiles to model each polynomial.

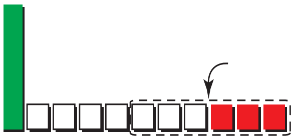
From  remove .

What other way could you model $(3x - 4) - (2x + 3)$?

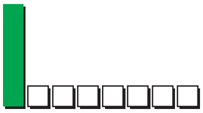
When you remove , you are left with  or .

You cannot yet remove 🟥🟥🟥 since there are no positive 1-tiles.

Add three zero pairs.



Now, you can remove 🟥🟥🟥.

You are left with  or $x - 7$.

$(3x - 4) - (2x + 3) = x - 7$

Recall that when you subtract integers, you can add the opposite.
$(-2) - (+3)$
$= (-2) + (-3)$
$= -5$

Method 2: Add the Opposite
One way to subtract a polynomial is to add the opposite terms.

$$\begin{aligned} &(3x - 4) - (2x + 3) \\ =\ &(3x - 4) + (-2x - 3) \\ =\ &3x - 4 - 2x - 3 \\ =\ &3x - 2x - 4 - 3 \\ =\ &1x - 7 \text{ or } x - 7 \end{aligned}$$

How do you know the opposite of $2x + 3$ is $-2x - 3$?

WWW Web Link

To learn more about adding and subtracting polynomials, go to www.mathlinks9.ca and follow the links.

Show You Know

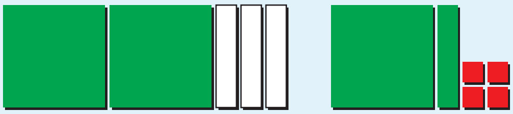
a) Simplify the following expression. Model your solution.
$(2x - 3) - (-x + 2)$

b) Subtract and combine like terms.
$(5x^2 - x + 4) - (2x^2 - 3x - 1)$

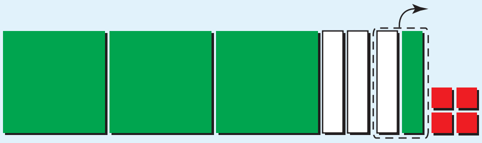
Key Ideas

- You can add or subtract polynomials. You can use models to help simplify the expression.

  $(2x^2 - 3x) + (x^2 + x + 4)$



  Group like terms. Remove any zero pairs.



  $(2x^2 - 3x) + (x^2 + x + 4) = 3x^2 - 2x + 4$

- The opposite of a polynomial is found by taking the opposite of each of its terms.

  The opposite of $-3x^2 + x + 1$ is $3x^2 - x - 1$.

- To subtract a polynomial, you can add the opposite terms.

$$\begin{aligned}
&(6x^2 - 3x + 4) - (x^2 - 3x + 2)\\
&= (6x^2 - 3x + 4) + (-x^2 + 3x - 2)\\
&= 6x^2 - x^2 - 3x + 3x + 4 - 2\\
&= 5x^2 + 0x + 2\\
&= 5x^2 + 2
\end{aligned}$$

$-3x + 3x = 0x$ or 0, so it does not need to be included in the answer.

Check Your Understanding

Communicate the Ideas

1. Jeanette and Tim find the answer to $(3x^2 - 5x) - (4 - 2x)$. Jeanette claims the simplified answer has three terms. Tim says it has only two terms. Who is correct? How do you know?

2. What is the opposite of $-x^2 + 2x - 3$? Use diagrams and then use symbols to determine the answer. Which method do you prefer? Why?

3. Identify any errors in Mei's work and correct them.

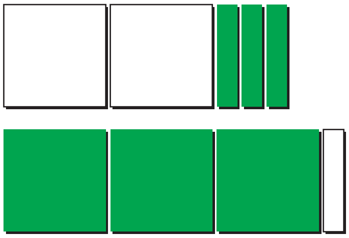
$$\begin{aligned}
&(-2x^2 + 7) - (3x^2 + x - 5)\\
&= (-2x^2 + 7) + (-3x^2 - x + 5)\\
&= -2x^2 - 3x^2 - x + 7 + 5\\
&= 5x^2 - x + 12
\end{aligned}$$

4. Create a situation in which the polynomials $3x + 2$ and $5x - 1$ are involved. In your situation, what does $(3x + 2) + (5x - 1)$ represent?

Practise

For help with #5 to #7, refer to Example 1 on page 191.

5. Which addition statement does the diagram model?



A $(2x^2 - 3x) + (3x^2 - x)$

B $(-2x^2 + 3x) + (3x^2 + x)$

C $(-2x^2 + 3x) + (3x^2 - x)$

6. Add the polynomials.

a) $(3x - 4) + (2x - 3)$

b) $(-a^2 - 3a + 2) + (-4a^2 + 2a)$

c) $(5p + 5) + (5p - 5)$

d) $(2y^2 - 15) + (6y + 9)$

7. Perform the indicated operation and simplify by combining like terms.

a) $(-3x + 4) + (6x)$

b) $(3n - 4) + (7 - 4n)$

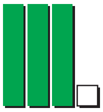
c) $(2b^2 - 3) + (-b^2 + 2)$

d) $(5a^2 - 3a + 2) + (-4a^2 + 2a - 3)$

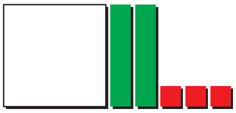
For help with #8 to #12, refer to Example 2 on pages 192–193.

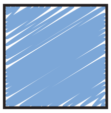
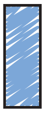
8. What is the opposite of the expression represented by each diagram? Express your answer using both diagrams and symbols.

a)

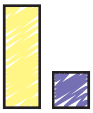


b)

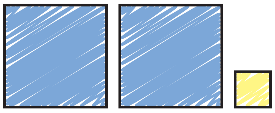


9. Let  represent x^2,  represent x, and represent 1. The same diagrams in yellow represent negative quantities Determine the opposite of the expression represented by each diagram. Use both diagrams and symbols to express your answer.

a)



b)



10. What is the opposite of each expression?

a) $-9x$

b) $5d + 6$

c) $-2x^2 + 3x - 5$

11. What is the opposite of each expression?

a) $3x - 7$

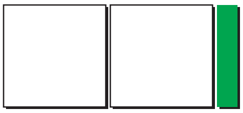
b) $4g^2 - 4g + 2.5$

c) $v^2 + 8v - 1$

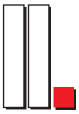
12. Which of the following represents the opposite of $2x^2 - x$?

A $-2x^2 - x$

B



C



D $2x^2 + x$

For help with #13 to #15, refer to Example 3 on page 194.

13. Draw a diagram to model the subtraction statement $(-3x^2 + 4x) - (-2x^2 - x)$.

14. Simplify by combining like terms.

a) $(2x - 3) - (5x - 1)$

b) $(-3b^2 - 5b) - (2b^2 + 4b)$

c) $(5 - 6w) - (-2 - 3w)$

d) $(m + 7) - (m^2 + 7)$

15. Subtract.

a) $(8c - 3) - (-5c)$

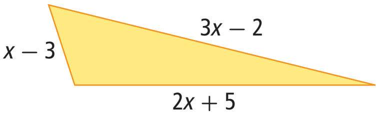
b) $(-3r^2 - 5r - 2) - (r^2 - 2r + 4)$

c) $(y^2 - 5y) - (2y - y^2)$

d) $(6j^2 - 4j + 3) - (-2j^2 - 5)$

Apply

16. A triangle has the dimensions shown.

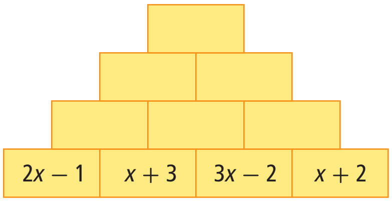


a) What does $(x - 3) + (3x - 2) + (2x + 5)$ represent?

b) Simplify the expression in part a).

c) If x has a value of 5, what is the perimeter of the triangle? Did you use the expression in part a) or part b) to find this answer? Why?

17. Complete the addition pyramid. Find the value in any box by adding the expressions in the two boxes immediately below it.



18. In Langley, British Columbia, you can rent a backhoe for \$399 per day and a bulldozer for \$550 per day. It costs \$160, round trip to move each piece of equipment back and forth to the job site.

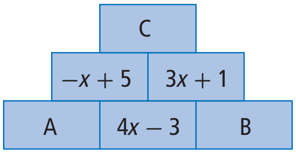
a) Write an expression for the total cost of renting the backhoe, before tax. Include transportation to and from the job site. What does your variable represent?

b) What is an expression for the total cost of renting and moving the bulldozer? Use the same variable as in part a).

c) What is an expression for the cost of renting both a backhoe and a bulldozer? Give your answer in its simplest form.

d) What is an expression for the difference in cost between renting the backhoe and the bulldozer? Give your answer in simplest form.

19. Consider the addition pyramid shown below.



a) Write an expression for box C. Do not simplify.

b) Show how you can find the value for boxes A and B. Simplify your answers.

20. The cost to print n copies of a book is $15n + 2000$ dollars. The cost to ship n copies of the book is $2n + 150$ dollars.

a) What is an expression for the total cost to print and ship n copies of the book?

b) What is the actual cost to print and ship 600 copies of the book?

c) What does $(15n + 2000) - (2n + 150)$ represent? Find a simpler expression for this subtraction statement.

21. Describe any errors in Jorge's work and how you would correct each one.

$$
\begin{aligned}
&(4p^2 - p + 3) - (p^2 + 3p - 2)\\
&= 4p^2 - p + 3 - p^2 - 3p - 2\\
&= (4p^2 - p^2) + (-p - 3p) + (3 - 2)\\
&= 3p^2 - 3p + 1
\end{aligned}
$$

22. Simplify by combining like terms.

a) $(6x - 7) + (3x - 1) + (x - 4)$

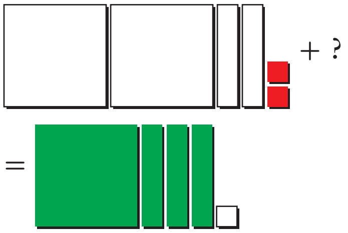
b) $(3a^2 - 4a) + (3a - 5) - (a^2 - 1)$

c) $(4t^2 - t + 6) - (t^2 + 2t - 4) + (2t^2 - 3t - 1)$

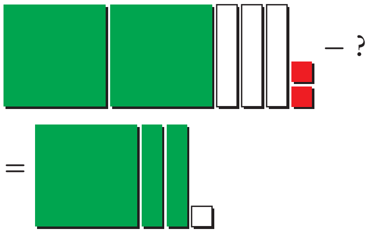
d) $(2x - 1.8) - (3.4x - 2.1) - (0.9x - 0.1)$

23. Replace each question mark with algebra tiles to make a true statement.

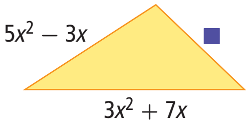
a)

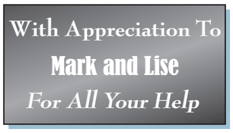


b)



24. The perimeter of the triangle shown is $12x^2 + 6x$, in metres. Find a polynomial representing the missing side length.





With Appreciation To
Mark and Lise
For All Your Help

25. Your student council plans to thank 25 students and staff who have made special contributions to the school. Wooden plaques cost \$17.95. It costs \$0.12 per letter to engrave a message on the plaque. All costs are before tax.

a) Write an expression for the cost of engraving 25 plaques with the following message and the name of your school. *Thank you for your generous support.*

b) Write an expression for the cost of buying and engraving the plaques.

c) Write an expression for the cost of buying and engraving 25 plaques with an unknown number of letters.

d) Show how to use the addition of polynomials to find the cost of 50 plaques if each plaque has the same number of letters and numbers.

Extend

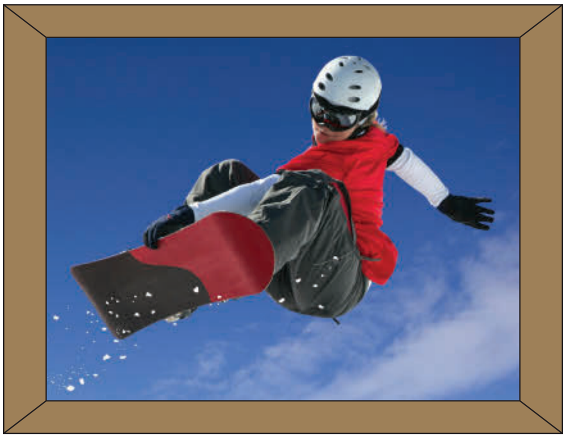
26. Kiesha's dad is a Haisla artist. He makes his own prints and sells them on the Internet. He will ship the prints to purchasers anywhere in Canada. For large prints, he charges \$30 to ship one print plus \$7 for each additional print. For small prints, he charges \$20 for one print plus \$5 for each additional print.

a) How much does her dad charge to ship two large prints?

b) How much does he charge to ship four small prints?

c) Write an expression to show how much he charges to ship an unknown number of large and small prints.

27. The length of the picture shown is 15 cm more than its width. The picture frame has a width of 4 cm. What is the minimum length of material needed to make the frame for this picture? Give your answer as a simplified expression.



28. A small manufacturer makes air quality monitoring kits for home use. The revenue, in dollars, from the sale of n kits can be shown by $-n^2 + 3600n$. The cost, in dollars, to make n kits is represented by $-3n^2 + 8600$. The manufacturer makes a profit if the cost subtracted from the revenue is positive.

a) Write an expression to find the profit. Simplify your answer.

b) Estimate and then calculate if the manufacturer will make a profit or suffer a loss after selling 20 test kits. Explain.

29. Simplify $(2x + 4x + 6x + 8x + \ldots + 2006x + 2008x) - (x + 3x + 5x + 7x + \ldots + 2005x + 2007x)$.

30. Mary is sewing two wall hangings. The length of one wall hanging is 56 cm greater than its width. The length of the other wall hanging is 15 cm greater than its width. Each of them has the same width. She is going to add a trim strip around each wall hanging. What is the total minimum length of trim she will need for both wall hangings?

Digital rights not available.

Digital rights not available.

Math Link

Try this number trick several times.

a) How can you find the original number from the number in the last step?

b) Use algebra to show why this number trick works.

c) Find or make your own number trick. Use algebra to show why it works.

Guess a Number

Step 1 Pick a number.

Step 2 Add 5.

Step 3 Double the sum.

Step 4 Subtract 10.

Chapter 5 Review

Key Words

For #1 to #6, choose the letter that best matches each description. You may use each letter more than once or not at all.

1. $3w$ is a like term

2. has three terms

3. monomial

4. opposite polynomial to $3x - 1$

5. polynomial of degree 2

6. contains the constant term 3

A $-3x + 1$

B $-4d + 3$

C $1 - 3x^2$

D $-w$

E $x - 6y + 2$

F $-3x - 1$

G $3f - 1$

5.1 The Language of Mathematics, pages 174–182

7. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a) $5 - p + px - p^2$

b) $3f - q$

c) $-2a$

d) $5xy - 27x^2 + 2$

8. What is the degree of each polynomial? Explain how you found your answers.

a) $6x^2$

b) $ab - 7a + 3$

c) $3 - y$

9. Provide an example of each of the following.

a) binomial

b) polynomial with three terms

c) polynomial of degree 2

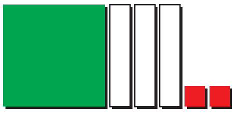
d) monomial that is a constant term

10. Model each expression.

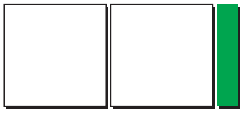
a) $1 - v$ **b)** $3x^2 - 2x + 1$

11. What expression is shown by each model?

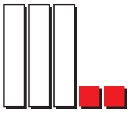
a)



b)



c)



12. You are selling used video games for \$10 and used books for \$4. The expression $10x + 4y$ provides a general statement of the value of your sales before tax.

DRAGON
$10
VIDEO GAME
HOME RUN
$4

a) What do the variables x and y represent?

b) How much money do you receive if you sell 6 video games and 11 books?

c) Write a new expression for selling DVDs at \$7.25 and CDs at \$5.

5.2 Equivalent Expressions, pages 182–189

13. Tom claims that 4 and $4x$ are like terms since they both contain 4. Do you agree? Explain.

14. For each expression, identify the coefficient, variables, and exponent for each variable.

a) $8xy^2$ **b)** $-c^2$ **c)** -4

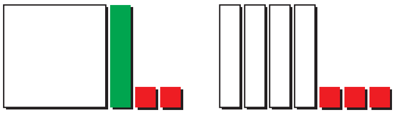
15. Identify the like terms in each group.

a) $7r \quad 3s \quad -s^2 \quad 7 \quad 4rs \quad -8s$

b) $-2x^2 \quad 3xy \quad x^2 \quad 5.3y \quad 2 \quad 3xy$

16. Explain how you can tell like terms by looking at them. Give four different sets of examples with at least three like terms in each set.

17. The following diagrams represent terms in an expression. Draw a new diagram with like terms together and write an expression for the simplified answer.



18. Use materials or diagrams to model each expression and show how to combine like terms. What is the simplified expression?

a) $3 - 2x + 1 + 5x$

b) $2x^2 - x + 4 - x^2 + 5x - 1$

19. Combine like terms.

a) $4a + 3 + 9a + 1$

b) $2b^2 - 5b - 4b^2 + 8b$

c) $1 - c + 4 + 2c - 3 + 6c$

20. Draw a shape with a perimeter represented by $(4x) + (3x - 1) + (x + 3) + (x - 2)$, where each quantity in parentheses represents the length of a side. Find a simpler expression for the perimeter by combining like terms.

21. Kara and Jasmine go to Splash-o-mania water park. The entrance fee is \$20.00. Kara rents a locker for \$1.50 per hour. Jasmine rents a tube for \$3.00 per hour.

a) What is an expression that represents the cost for Kara?

b) What is an expression that represents the cost for Jasmine?

c) What is a simplified expression for the total cost for both Kara and Jasmine to stay at the water park together for any number of hours?

5.3 Adding and Subtracting Polynomials, pages 190–199

22. a) Simplify each of the algebraic expressions.

$(4x - 3) + (x - 1)$ $(4x - 3) - (x - 1)$

b) How are the processes similar? How are they different?

23. Is $2x^2 - 3x$ the opposite of $3x - 2x^2$? Show how you know.

24. For each of the following expressions, what is the opposite?

a) -3 **b)** $7 - a$ **c)** $x^2 - 2x + 4$

25. a) Show how to simplify the following addition. Use two different methods.

$(3p + 4q - 9) + (2 - 5q - p)$

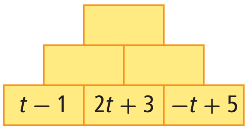
b) Which method do you prefer? Why?

26. Combine like terms.

a) $(-p + 7) + (4p - 5)$

b) $(a^2 - a - 2) - (5 - 3a^2 + 6a)$

27. Complete the subtraction pyramid. Find the value in any box by subtracting the two expressions in the boxes immediately below it. Subtract in order from left to right.



28. An end-of-year class party has a fixed cost of \$140 to cover printing, decorations, and awards. In addition, it costs \$12 to feed each person who attends.

a) What is an expression for the total cost of the party? What does your variable represent?

b) Create a short scenario to generate an addition or subtraction question with polynomials. Simplify by combining like terms.

Chapter 5 Practice Test

For #1 to #6, select the best answer.

1. Which polynomial is of degree 1?

A $3 - 7x$ **B** $xy - 1$

C $5x - 3xy$ **D** $x^2 - 5x + 2$

2. Which expression does not have zero as a constant term?

A $-5x$ **B** $k + 8$

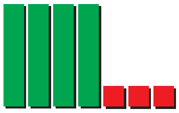
C $y^2 - 2y$ **D** $ab + b - c$

3. Which of the following is not equivalent to $3x - 5 + 2 - 7x$?

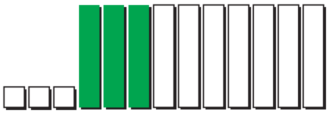
A $-4x - 3$

B $3x - 7x - 5 + 2$

C

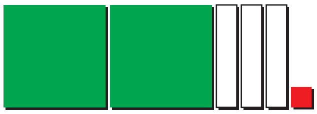


D

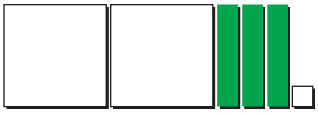


4. Which set of diagrams represents $3x - 2x^2 + 1$?

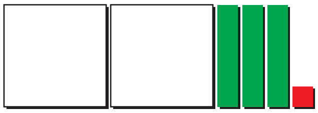
A



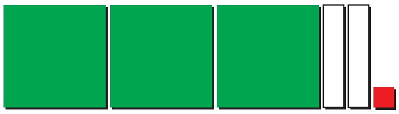
B



C



D



5. Which expression is a trinomial?

A abc^3

B $3mn$

C $ef + g^2$

D $-1 - x + c$

6. Which expression is the opposite of $-2k^2 + 3k - 1$?

A $-1 - 3k + 2k^2$

B $1 - 3k + 2k^2$

C $1 - 3k - 2k^2$

D $-1 - 3k - 2k^2$

Complete the statements in #7 and #8.

7. When you combine like terms, the expression $2t^2 - 5 - 8t^2 - 4$ becomes ▬.

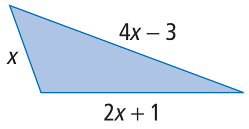
8. In the monomial $-q^2$, the value of the coefficient is ■.

Short Answer

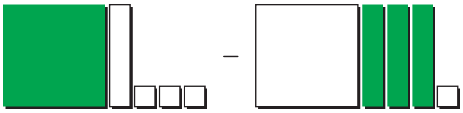
9. Draw a diagram to represent $x^2 - 2x$.

10. Create a single polynomial with
- two terms
- two variables
- degree 2
- a constant term

11. What is an expression, in simplest form, for the perimeter of the triangle?



12. Write an expression to represent what the diagrams show. Then, simplify.



13. Simplify. Use models for at least one of the expressions. Show your work.

a) $(2x^2 - 8x + 1) + (9x^2 + 4x - 1)$

b) $(4 - 6w) - (3 - 8w)$

Extended Response

14. The number of peanuts two squirrels bury can be represented by $4n + 7$ and $5n - 1$, respectively.

a) Write and simplify an expression for the number of peanuts both squirrels bury.

b) What could the expression $(5n - 1) - (4n + 7)$ represent?

c) What is a simpler expression for $(5n - 1) - (4n + 7)$?

15. The cost for a birthday party at Big Fun Bowling is \$100 for up to ten children, plus \$5 per pair of bowling shoes. To rent the party room, the cost is \$20, plus \$4 per child for pizza.

a) What is an expression for the cost of bowling for up to ten children?

b) What is an expression for the cost of pizza in the party room for up to ten children?

c) What is a simplified expression for the total cost of up to ten children going bowling and having pizza in the party room?

d) Estimate, then calculate, the cost of nine children going bowling and having pizza in the party room.

Math Link: Wrap It Up!

You are an illusionist who is about to amaze your audience with a number trick. However, before you try the trick, you need to know how it works.

a) Try the trick, Guess a Number, several times. What do you notice about the middle digit of the number in step 4?

b) What do you notice about the other two digits?

c) How does the information from parts a) and b) help you to understand this number trick?

d) Make up a number-guessing trick. Show how algebra can help explain your number trick.

Guess a Number

Step 1 Tell someone to write down a three-digit number with no repeating digits. During the entire trick, do not look at what the person writes.

Step 2 Have the person arrange the digits in decreasing order.

Step 3 Ask the person to arrange the same three digits in increasing order.

Step 4 Tell the person to subtract the number in step 3 from the number in step 2.

Step 5 Ask the person to circle one number in the difference.

Step 6 Ask what the other two digits are. Identify the digit that was circled.

Challenges

Kayaks for Rent

You are the owner of a kayak rental shop. You offer 20 single-person kayaks and 10 two-person kayaks. You rent single-person kayaks for \$35/day and two-person kayaks for \$45/day.

The average cost to you for maintenance is \$12/day for single-person kayaks and \$18/day for two-person kayaks.

1. Develop an expression that can be used to calculate the maximum profit that you can generate over any number of days. Identify your variables.
2. What is your maximum profit for a 60-day period?
3. High demand allows you to increase your prices. You realize that you can make the same profit in #2 in 45 days. What changes would you make to your rental prices? Justify your new pricing structure.
4. You would like to expand your business and plan to buy some canoes and tents. However, you need to confirm your business plan first. You will charge \$40/day for a canoe and \$50/day for a large tent. You want to make a profit of at least \$20 000/month from the canoes and tents. It costs you \$17/day for maintenance and loan payments for a canoe, and \$13/day for a tent. You have not yet decided how many of each you will buy. Develop an expression that allows you to meet your profit goal. Assume there are 30 days in a month.

What Have You Got to Hide?

Prime numbers have fascinated mathematicians for thousands of years. The earliest known reference to prime numbers dates to 500 B.C.E. At that time Pythagoras' school of Greek mathematicians discovered prime numbers and realized that these are the building blocks of whole numbers.

Prime numbers may be used to create encrypted data such as that used to hide personal information being sent on the internet. You be the cryptographer!

The following polynomials can be used to create prime numbers:

- $n^2 - n + 41$
- $n^2 + n - 1$
- $n^2 - n + 17$

Your challenge is to investigate whether the addition and/or subtraction of these polynomials can create prime numbers.

1. Test each polynomial using at least three possible n values. Show your testing process and the results of your test.
2. If decimals or fractions are substituted for n, are your results prime numbers? Show your testing process and results. What is your conclusion?
3. Add and/or subtract several combinations of the given polynomials. Test the resulting polynomials to see if they generate prime numbers when values are substituted in. Show your work.