CHAPTER

6

Linear Relations

Some dimes have an image of *Bluenose*. This famous Canadian schooner is a symbol of Nova Scotia's prominence in the fishing industry and international trade. *Bluenose* was launched in 1921 in Lunenburg, Nova Scotia, as a racing ship and a fishing schooner.

A replica schooner, *Bluenose II*, has served as the floating ambassador for Nova Scotia since 1971.

The sailors who raced *Bluenose* had to think about factors such as length of a race, wind speed, and current. In racing, one factor is often related to another. For example, it takes longer to run a race if the current is strong. Sometimes the relationship between two factors can be represented mathematically in a linear relation.

Shipbuilders and sailors today continue to use linear relations to design, build, and operate vessels such as windsurf boards, racing sailboats, and supertankers.

What You Will Learn

- to represent pictorial, oral, and written patterns using linear expressions, equations, and graphs
- to interpret patterns in graphs
- to solve problems involving pictorial, oral, and written patterns by using linear equations and graphs

Did You Know?

From 1921 to 1938, *Bluenose* won the annual International Fishermen's Trophy for racing ships. After World War II, fishing schooners were retired. The undefeated *Bluenose* was sold to operate as a freighter in the West Indies. The ship sank in 1946.

Key Words

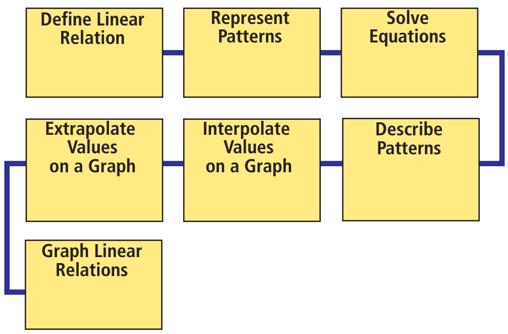
linear relation
linear equation
numerical coefficient
variables
constant
interpolate
extrapolate

Literacy Link

A sequence chart can help you review terms and understand the order of concepts in this chapter. The chart is designed to be used throughout the chapter to help you connect new concepts.

Create a sequence chart in your math journal or notebook. As you work through the chapter, complete the chart.

- In the first box, define the term *linear relation*.
- In the second box, list the ways you learned to represent patterns.
- In the third box, show an example of how to solve a linear equation.
- In the fourth box, show how to describe a pattern using a linear equation.
- In the fifth box, explain how to interpolate values on a graph.
- In the sixth box, explain how to extrapolate values on a graph.
- In the seventh box, describe how to create a graph from a linear equation.



FOLDABLES™
Study Tool

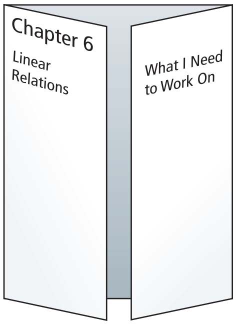
Making the Foldable

Materials

- sheet of 11 × 17 paper
- ruler
- three sheets of 8.5 × 11 grid paper
- stapler
- sheet of 8.5 × 11 paper
- scissors

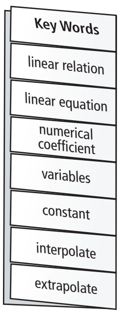
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Label it as shown.



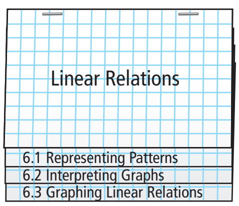
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. On half of the sheet, cut a straight line across the width every 3.5 cm, forming eight tabs. Label the tabs as shown.



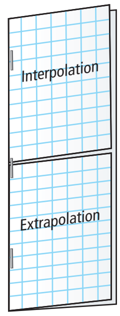
Step 3

Stack two sheets of 8.5 × 11 grid paper so that the bottom edges are 2.5 cm apart. Fold the top edge of the sheets and align the edges so that all tabs are the same size. Staple along the fold. Label as shown.



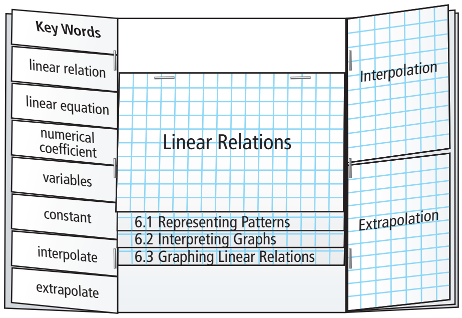
Step 4

Fold the short side of a sheet of 8.5 × 11 grid paper in half. Fold in two the opposite way. Make a cut through one thickness of paper, forming two tabs. Label the tabs as shown.



Step 5

Staple the booklets you made from Steps 2, 3, and 4 into the Foldable from Step 1, as shown.



Using the Foldable

As you work through Chapter 6, define the Key Words, and record notes, examples, and Key Ideas in the appropriate section. Use the grid paper to show examples of interpolation and extrapolation.

Use the back of the Foldable to record your ideas for the Math Link: Wrap It Up!

On the front of the Foldable, keep track of what you need to work on. Check off each item as you deal with it.

Math Link

Marine Travel

You may never get to see a supertanker, even if you live on the coast, because they are too large to enter most ports. Supertankers can weigh 200 000 to 400 000 tonnes. The top speed of a supertanker when carrying a full load is only about 30 km/h. Once they are moving, they are hard to stop.

A crash stop manoeuvre from "full ahead" to "full reverse" can stop a loaded supertanker in about 15 min within approximately 3 km. The table of values shows the speed of a supertanker during a crash stop.

Time, *t* (min)	Speed, *s* (km/h)
0	30
3	24
6	18
9	12
12	6
15	0

1. a) What do you think the speed will be at 4 min? 5 min?
   b) Describe the pattern you see in the data. What do you notice about how the values change from one set of coordinate pairs to the next?
2. a) On a graph, plot the coordinate pairs in the table.
   b) Which variable did you plot on the horizontal axis? Why did you select that variable?
   c) Which variable did you plot on the vertical axis? Why did you select that variable?
   d) Does the graph match your pattern description? Explain.
3. Write an equation to model the data on the graph.
4. a) What value did you choose for the numerical coefficient and the constant?
   b) How did you determine each value?
5. A smaller tanker can stop in less time. The equation $s = -3t + 30$, where speed, s, in km/h, and time, t, in min, models stopping the smaller tanker.
   a) What would be the speed of the tanker at 7 min? How did you determine your answer?
   b) How much time would it take for the tanker to slow down to 8 km/h? Compare your solution with that of a classmate. Explain how you arrived at your answer.

In this chapter, you will explore mathematical relationships between variables, such as time and distance for different types of boats. At the end of the chapter, you will research and plan an adventure trip on water, showing mathematical relationships between variables.

Literacy Link

A metric tonne (t) is a measurement of mass that equals 1000 kg.

WWW Web Link

For more information about supertankers and other oil tankers, go to www.mathlinks9.ca and follow the links.

6.1 Representing Patterns

Focus on...

After this lesson, you will be able to...

- represent pictorial, oral, and written patterns with linear equations
- describe contexts for given linear equations
- solve problems that involve pictorial, oral, and written patterns using a linear equation
- verify linear equations by substituting values

A skiff is a two-person sailing boat that can be used for racing. The carbon foam sandwich hull and multiple sails allow the boat to travel at speeds of 5 to 35 knots.

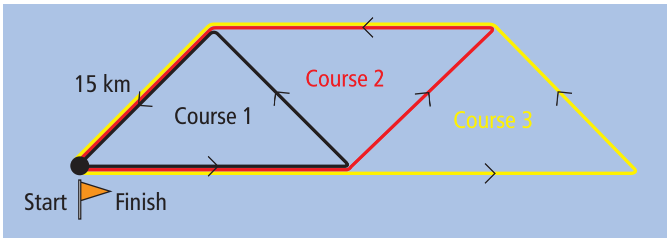
Did You Know?

A *knot* is a measure of a boat's speed. One knot is equal to 1.852 km/h. The term comes from the time when sailors measured the speed of a ship by tying knots an equal distance apart on a rope. The rope was gradually let out over the back of the ship at the same time that an hourglass was tipped. The sailors counted the number of knots that were let out until the sand ran out in the hourglass.

Materials

- ruler
- coloured pencils

Explore Patterns



The first three racing courses are shown for a class of skiffs. Each leg of the course is 15 km.

How could you determine the total distance of each racing course? Describe different strategies you could use to solve this problem.

1. Draw what you think the next two courses might look like.

2. Organize the information for the first five courses so that you can summarize the results.

3. a) Describe the pattern in the race course lengths. Then, check that your Courses 4 and 5 fit the pattern.
   b) Describe the relationship between the course number and the length of the course.

4. Write an equation that can be used to model the length of the course in terms of the course number. Explain what your variables represent.

Reflect and Check

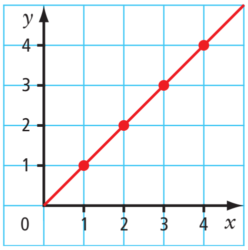
5. a) What are two methods you could use to determine the length of Course 9?
   b) Which method do you prefer? Explain why.

6. a) Determine which course is 135 km long.
   b) Determine the length of Course 23.
   c) How did you determine the answers to parts a) and b)?
   d) Discuss your solutions with a classmate.

Link the Ideas

Literacy Link

A *linear relation* is a relation that appears as a straight line when graphed.



A *linear equation* is an equation whose graph is a straight line.

Example 1: Describe a Pictorial Pattern Using a Linear Equation

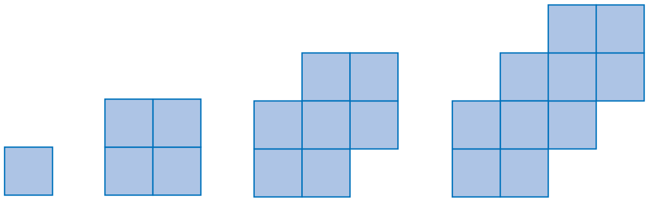


Figure 1 Figure 2 Figure 3 Figure 4

a) Describe the pattern.

b) Create a table of values to represent the linear relation between the number of squares and the figure number for the first four figures.

c) Write a linear equation to represent this pattern.

d) How many squares are in Figure 12?

e) Which figure number has 106 squares? Verify your answer.

Solution

a) The pattern is increasing. Each figure has three more squares than the previous figure. The squares have been added to the upper right corner of the previous pattern.

Strategies

Look for a Pattern

b)

Figure Number, *n*	Number of Squares, *s*
1	1
2	4
3	7
4	10

Tech Link

You can use a spreadsheet program to create a table.

c) Add two columns to the table to help determine the pattern.

Figure Number, *n*	Number of Squares, *s*	Pattern	
		Multiply *n* by 3	Subtract 2 From Result
1	1	3	1
2	4	6	4
3	7	9	7
4	10	12	10

The number of squares, *s*, increases by 3 for each figure number, *n*. Multiplying the figure number, *n*, by 3 results in 2 more than the number of squares. Therefore, subtracting 2 from 3*n* equals the number of squares, *s*.

The equation is $s = 3n - 2$.

Literacy Link

In the equation $s = 3n - 2$,

- the *numerical coefficient* is 3
- the *variables* are *s* and *n*
- the *constant* is −2

d) Substitute $n = 12$ into the equation and solve for s.

$$\begin{aligned} s &= 3(12) - 2 \\ &= 36 - 2 \\ &= 34 \end{aligned}$$

There are 34 squares in Figure 12.

e) Substitute $s = 106$ into the equation and solve for n.

$$\begin{aligned} 106 &= 3n - 2 \\ 106 + 2 &= 3n - 2 + 2 \\ 108 &= 3n \\ \frac{108}{3} &= \frac{3n}{3} \\ 36 &= n \end{aligned}$$

The solution is $n = 36$.

How else could you solve the problem?

Check:

Left Side = 106 Right Side $= 3n - 2$

$= 3(36) - 2$

$= 108 - 2$

$= 106$

Left Side = Right Side

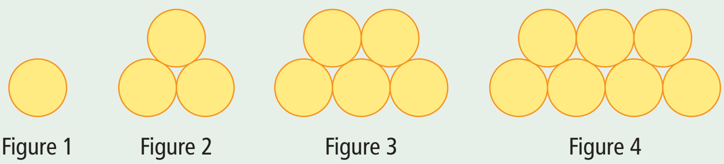
The solution is correct. Figure 36 has 106 squares.

Show You Know

Strategies

Use a Variable

a) Write an equation to represent the number of circles in relation to the figure number.



b) How many circles are in Figure 71? Explain how you determined the answer.

c) Which figure number has 83 circles? How did you arrive at your answer?

Example 2: Describe a Written Pattern Using a Linear Equation

A bead design for a necklace has an arc shape:

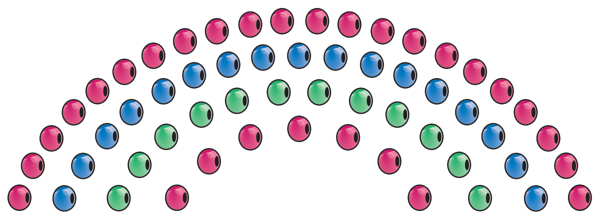
- Row 1 has seven red beads.
- Row 2 has five additional beads and all the beads are green.
- Row 3 has five additional beads and all the beads are blue.
- The pattern repeats. Five beads are added to each successive row.

a) Draw the pattern for the first four rows.
b) Make a table of values showing the number of beads in relation to the row number.
c) What equation shows the pattern between the row number and the number of beads in the row?
d) How many beads are in Row 4? Explain how to check your answer.
e) How many beads are in Row 38?
f) If the bead pattern were continued, which row number would have 92 beads? How did you determine the answer?

Solution

Strategies
Draw a Diagram

a)



Strategies
Make an Organized List or Table

b)

Row Number, *n*	Number of Beads, *b*
1	7
2	12
3	17
4	22

c) Add two columns to the table to help determine the pattern.

Row Number, *n*	Number of Beads, *b*	Pattern	
		Multiply *n* by 5	Add 2 to Result
1	7	5	7
2	12	10	12
3	17	15	17
4	22	20	22

The equation is $b = 5n + 2$.

Look at the diagram of the pattern. In the equation, what does the 5 mean? What does the 2 mean?

d) Count the number of beads in Row 4. There are 22 beads. You can check this by substituting $n = 4$ into the equation and solving for b.

$$\begin{aligned} b &= 5n + 2 \\ &= 5(4) + 2 \\ &= 20 + 2 \\ &= 22 \end{aligned}$$

There are 22 beads in Row 4.

e) Substitute $n = 38$ into the equation and solve for b.

$$\begin{aligned} b &= 5n + 2 \\ &= 5(38) + 2 \\ &= 190 + 2 \\ &= 192 \end{aligned}$$

There are 192 beads in Row 38.

How else could you solve the problem?

What colour are the beads in Row 38? Explain how you know.

f) Substitute $b = 92$ into the equation and solve for n.

$$\begin{aligned} 92 &= 5n + 2 \\ 92 - 2 &= 5n + 2 - 2 \\ 90 &= 5n \\ \frac{90}{5} &= \frac{5n}{5} \\ 18 &= n \end{aligned}$$

The solution is $n = 18$.

Check:

Left Side = 92   Right Side $= 5n + 2$

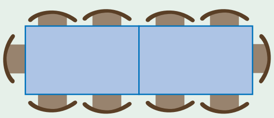
$$\begin{aligned} &= 5(18) + 2 \\ &= 90 + 2 \\ &= 92 \end{aligned}$$

Left Side = Right Side

The solution is correct. Row 18 has 92 beads.

Show You Know

In a banquet hall, a single rectangular table seats six people. Tables can be connected end to end as shown. Four additional people can be seated at each additional table of the same size.



a) What linear equation could represent this situation? Share with a classmate how you determined the equation.

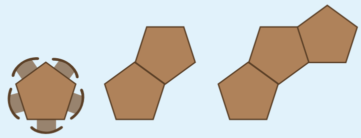
b) How many tables connected together will seat 26 people?

Key Ideas

- Many pictorial and written patterns can be represented using a table of values or a linear equation.

  The pentagonal table can seat five people. The tables can be connected to form longer tables.

Number of Tables, *t*	Number of Sides, *s*	Pattern: Multiply *t* by 3 and Add 2
1	5	5
2	8	8
3	11	11



  The equation that models the pattern is $s = 3t + 2$.

- Linear equations can be verified by substituting values.

  Substitute $t = 3$ into the equation:

  $s = 3(3) + 2$

  $= 9 + 2$

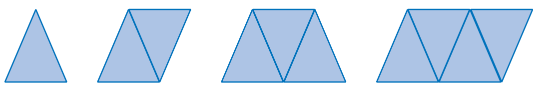
  $= 11$

  The calculated value matches the value in the table.

Check Your Understanding

Communicate the Ideas

1. a) Explain how to develop a linear equation to represent this pattern.



b) What is the equation? Explain what each variable represents.

c) Compare your equation with one of a classmate's.

2. Christina and Liam work in a shoe store and earn a flat rate of $35/day plus $6.25 for every pair of shoes they sell. Each got a different value for how much they would earn after selling eight pairs of shoes.

Christina: I substituted $p = 8$ into the equation $w = 6.25p + 35$. When I solved for *w*, I got $85.

Liam: I substituted $p = 8$ into the equation $w = 6.25p$. When I solved for *w*, I got $50.

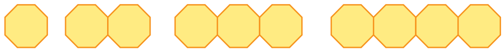
Who is correct? Explain how you know. What mistake did the other person make?

3. Describe to a partner how you could determine the ninth value in the following number pattern: 4, 7.5, 11, 14.5, 18, … .

Practise

For help with #4 to #6, refer to Example 1 on pages 212–213.

4. a) Describe the relationship between the number of regular octagons and the number of sides in this pattern.



b) Make a table of values showing the number of sides for each figure in relation to the number of octagons.

c) Write an equation to model the number of sides of each shape. Explain what each variable represents.

d) How many sides would a shape made up of 17 octagons have?

e) How many octagons are needed to make a figure with 722 sides?

5. a) Make a table of values to show the number of circles in relation to the figure number.

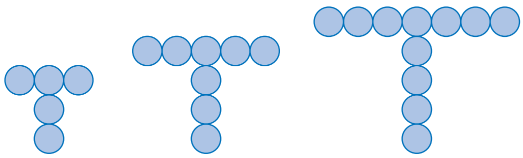


Figure 1 Figure 2 Figure 3

b) Describe the relationship between the number of circles and the figure number.

c) Develop an equation that can be used to determine the number of circles in each figure. Explain what each variable represents.

d) How many circles are in Figure 17?

e) Which figure number has 110 circles?

6. Laura used green and white tiles to create a pattern.

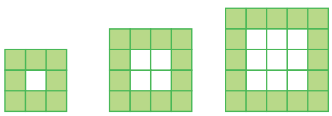


Figure 1 Figure 2 Figure 3

a) Make a table of values to show the number of green tiles in relation to the figure number.

b) Describe the relationship between the number of green tiles and the figure number.

c) Develop an equation to model the number of green tiles. Explain what each variable represents.

d) How many green tiles are in Figure 24?

e) Which figure number has 176 green tiles? Verify your answer.

For help with #7 to #9, refer to Example 2 on pages 214–215.

7. Matt created the following number pattern: 7, 16, 25, … .

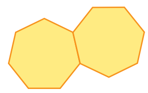
a) Make a table of values for the first five terms.

b) Develop an equation that can be used to determine the value of each term in the number pattern.

c) What is the value of the 123rd term?

d) Which term has a value of 358?

8. The figure shows two regular heptagons connected along one side. Each successive figure has one additional heptagon. Each side length is 1 cm.



a) Draw the first six figures. Then, describe the pattern.

b) Make a table of values showing the perimeter for the first six figures.

c) What equation can be used to determine the perimeter of each figure? Identify each variable.

d) What is the perimeter of Figure 12?

e) How many heptagons are needed to create a figure with a perimeter of 117 cm?

Literacy Link

A regular heptagon has seven sides of equal length.

9. Jessica created a number pattern that starts with the term −5. Each subsequent number is 3 less than the previous number.

a) Make a table of values for the first five numbers in the pattern.

b) What equation can be used to determine each number in the pattern? Verify your answer by substituting a known value into your equation.

c) What is the value of the 49th term?

d) Which term has a value of −119? Verify your answer.

10. What linear equation models the relationship between the numbers in each table?

a)

x	y
0	13
1	16
2	19
3	22

b)

r	p
0	17
1	24
2	31
3	38

c)

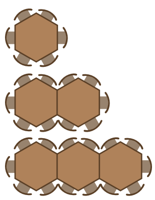
k	t
1	−1.3
2	1.4
3	4.1
4	6.8

d)

f	w
1	−0.5
2	−4
3	−7.5
4	−11

Apply

11. Rob is in charge of arranging hexagonal tables for a parent-night presentation. The tables, which can seat six people, can be connected to form longer tables.



a) Develop an equation to model the pattern. Identify each variable.

b) How many parents can be seated at a row of five tables?

c) Check your answer for part b). Show your work.

d) A group of 30 people want to sit together. How many tables must be joined together to seat them?

12. A school pays a company $125 to design gym T-shirts. It costs an additional $15 to make each T-shirt.

a) Copy and complete the table of values.

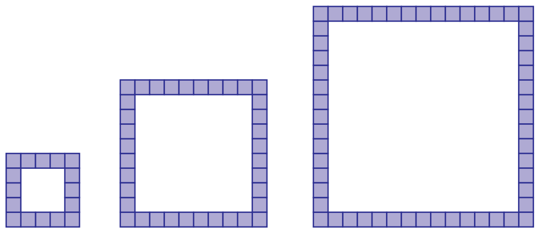
Number of T-Shirts	Cost ($)
0	125
5	200
10	
15	
35	
	950

b) Develop an equation to determine the cost of the T-shirts. Explain the meaning of the numerical coefficient.

c) What would it cost to make 378 T-shirts?

d) If the school store has a budget of $2345 for T-shirts, how many T-shirts can be ordered?

13. An art store sells square picture frames with a border of tiles that each measure 2 cm by 2 cm. The smallest frame is 10 cm by 10 cm and requires 16 tiles.

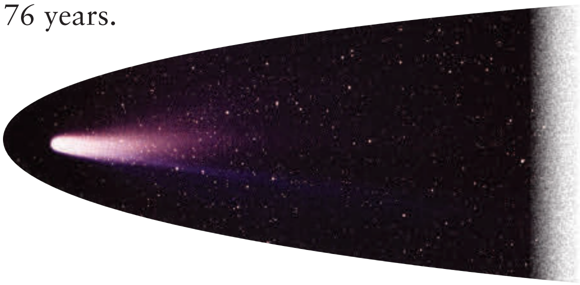


a) Develop an equation to determine the number of tiles required for each size of frame.

b) How many tiles are needed to make a frame that is 30 cm by 30 cm?

c) What are the dimensions of a square frame made with 196 tiles?

14. Edmund Halley, after whom Halley's comet was named, predicted that the comet would appear in 1758. The comet appears approximately every 76 years.



a) Use a table to show the years of the next six sightings after 1758.

b) When will Halley's comet appear in your lifetime?

c) Write an equation that can be used to predict the years when Halley's comet will appear.

d) Will Halley's comet appear in the year 2370? How did you arrive at your answer?

Science Link

A comet, which is made of frozen gas and dust, orbits around the sun. The dust tail of a comet can be up to 10 000 000 km long. This is 2.5 times as great as the average distance from Earth to the moon.

Extend

15. Find the pattern that expresses all the numbers that are 1 more than a multiple of 3.

a) What is the 42nd number?

b) How can your pattern test to see if 45 678 is 1 more than a multiple of 3?

16. a) A landscaper is planting elm trees along a street in a new subdivision. If elm trees need to be spaced 4.5 m apart, then how long is a row of n elm trees? Write the equation.

b) The street is 100 m long. If the landscaper wants to line the street on both sides with elm trees, how many trees will be needed? Will the trees be evenly spaced along the entire street?

17. A ball is dropped from a height of 2 m. The ball rebounds to $\frac{2}{3}$ of the height it was dropped from. Each subsequent rebound is $\frac{2}{3}$ of the height of the previous one.

a) Make a table of values for the first five rebound heights in the pattern.

b) What is the height of the fourth rebound bounce?

c) Is this a linear relation? Explain how you know.

Math Link

You are in charge of developing a racing course for a sailboat race on Lake Diefenbaker, in Saskatchewan. Five classes of sailboats will race on courses that are the same shape, but different lengths.

a) Design a racing course based on a regular polygon. The shortest course must be at least 5 km long. The longest course must be no longer than 35 km.
- Draw and label a diagram of the racing course. Show at least the first four courses. Record the total length of each course.

b) Develop a linear relation related to your racing course.
- Make a table of values.
- Develop a linear equation that represents the relationship between the course number and the course distance.

c) Develop a problem related to your racing course. Provide the solution and verify it.

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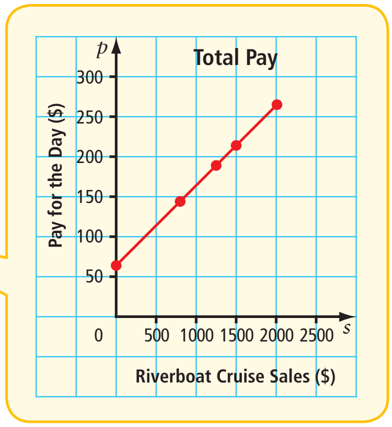
6.2 Interpreting Graphs

Focus on...

After this lesson, you will be able to...

- **describe patterns found in graphs**
- **extend graphs to determine an unknown value**
- **estimate values between known values on a graph**
- **estimate values beyond known values on a graph**



Richard is paid a daily salary and commission to sell riverboat cruises.

As a salesperson, he recognizes that it is important to know his sales in order to determine his commission. He is considering using a graph to calculate this amount. What do the variables *p* and *s* represent? What other information does the graph provide?

Literacy Link

Commission is a form of payment for services. Salespeople who earn commissions are paid a percent of their total sales. For example, a clothing store might pay a commission of 10% of sales.

Explore Using a Graph to Solve Problems

Richard is paid $64/day plus a commission of 10% of sales for selling riverboat cruises.

1. The table provides some information about his sales and daily pay.

Day	River Cruises Sold ($)	Pay for the Day ($)
Monday	1500	214
Tuesday	1250	
Wednesday	800	
Thursday	0	64
Friday	2010	

a) On the graph, locate the data points for each day.

b) How could you use the graph to estimate the missing values on the table? Try your method.

Literacy Link

On the graph, the line joining the points shows that the data are *continuous*. This means that it is reasonable to have values between the given data points.

2. a) Estimate how much Richard must sell to earn \$300 in one day. Describe your strategy.

b) Estimate how much Richard would earn if he had sales of \$1150. Describe your strategy.

3. The graph represents the linear equation $p = \frac{s}{10} + 64$. How could you use this information to determine the answers to #2? Explain why your strategy is effective.

Reflect and Check

4. a) Explain how your graph helped you to answer #2.

b) Discuss your strategies with a classmate.

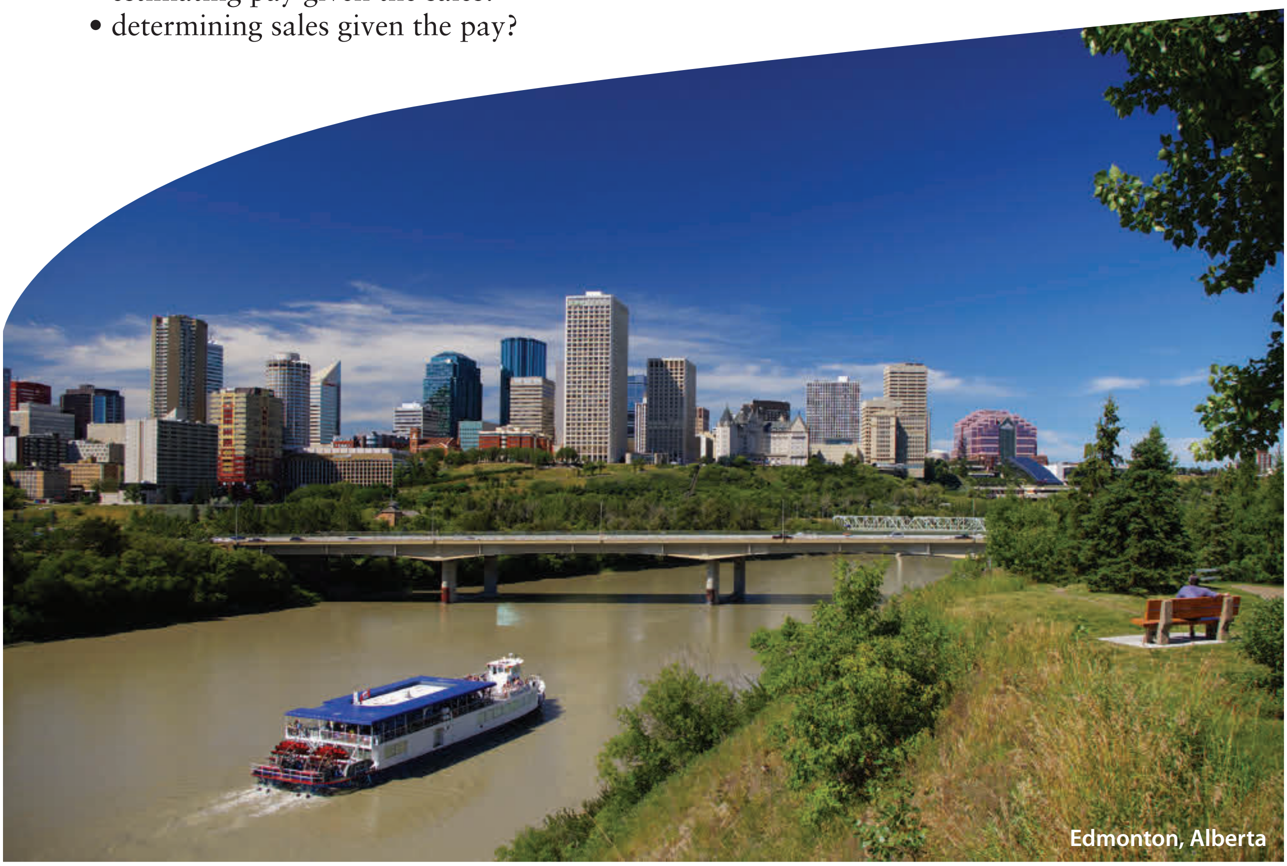
5. Work with a partner and use your graph.

a) How much would Richard earn if he had sales of \$2400?

b) How much must Richard sell to earn \$175 in one day?

6. a) List an advantage and a disadvantage of using a graph and an equation to determine values.

b) Which method is more effective when
- estimating pay given the sales?
- determining sales given the pay?


Edmonton, Alberta

Link the Ideas

interpolate
- estimate a value between two given values
- interpolation should be used only when it makes sense to have values between given values. For example, 5.4 people does not make sense.

Literacy Link

When describing variables on a graph, express the y-variable in terms of the x-variable. For example, the graph shows the relationship between temperature and altitude.

Tech Link

You can use a spreadsheet program to create the graph.

Strategies

Estimate and Check

Example 1: Solve a Problem Using Interpolation

A weather balloon recorded the air temperature at different altitudes. The data approximately represent a linear relationship.

Altitude, *a* (m)	350	750	1000	1500	1800
Temperature, *t* (°C)	11.4	5.7	2.1	−5.0	−10.0

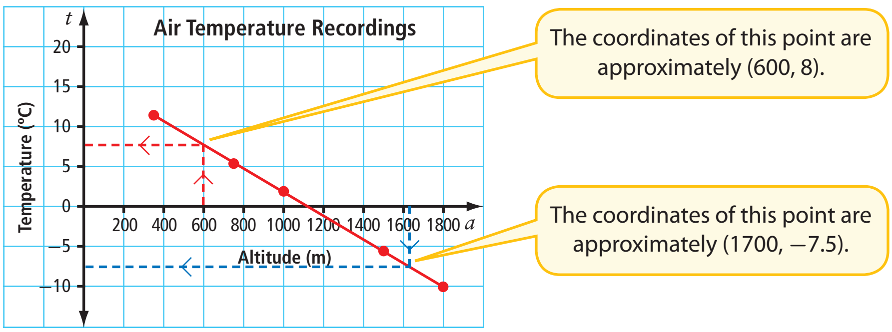
a) Interpolate the approximate value for the air temperature when the balloon was at a height of 600 m.

b) What was the approximate altitude of the balloon at an air temperature of −7.5 °C?

c) Is it possible to interpolate the precise value for the air temperature when the altitude is 1050.92 m? Explain.

Solution

Graph the data. Since temperature change is continuous, you can draw a straight line to connect the data points.



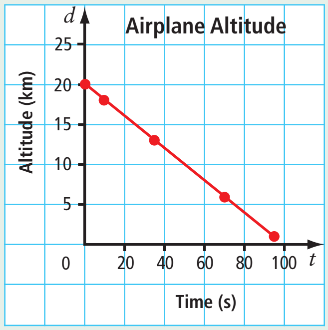
a) On the graph, draw a vertical line from 600 to the point where the line intersects the graph. From the intersection point, draw a horizontal line to the y-axis. The value where the horizontal line meets the y-axis is approximately 8.

b) On the graph, draw a horizontal line from −7.5 to the point where the line intersects the graph. From the intersection point, draw a vertical line to the x-axis. The value where the vertical line meets the x-axis is approximately 1700.

c) No, you cannot interpolate a precise value. The temperature is related to altitude values to the nearest 50 m. There is too much uncertainty to accurately predict temperatures based on altitude measurements that have a precision of $\frac{1}{100}$ of a metre. You could, however, estimate the temperature for an altitude of between 1000 m and 1100 m.

Show You Know

This graph shows a plane's altitude as it lands. The relationship between altitude and time is approximately linear.



a) What was the plane's approximate altitude at 50 s?

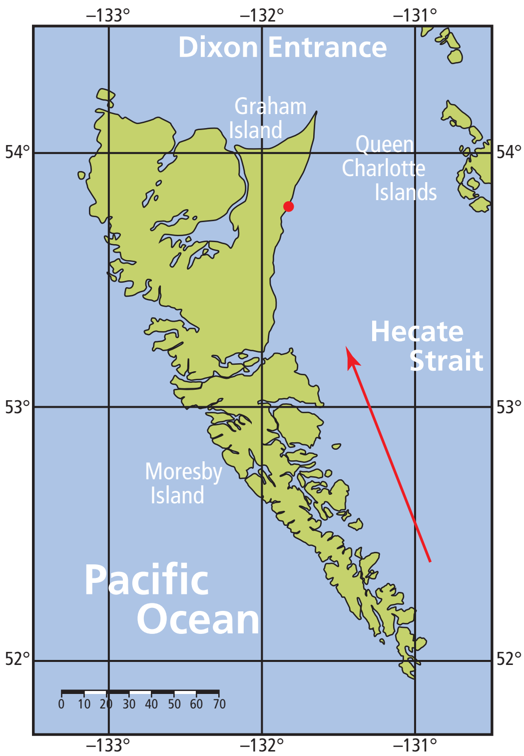
b) At what time was the plane's altitude approximately 11 km?

c) Is it appropriate to join the points with a straight line? Explain.

Example 2: Solve a Problem Using Extrapolation

Anna is kayaking up the east coast of the Queen Charlotte Islands toward Graham Island.

Anna's course is shown by the red arrow on the map.



a) If Anna continues on her present course, **extrapolate** the values of the coordinates for latitude and longitude where she will land.

b) Could you use extrapolation to estimate where Anna sailed from? Explain.

extrapolate
- estimate a value beyond a given set of values
- extrapolation should be used only when it makes sense to have values beyond given values

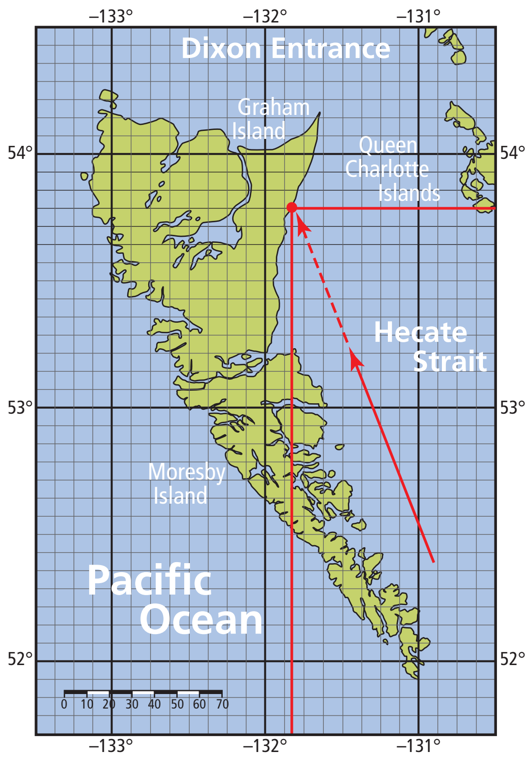
Did You Know?

The Queen Charlotte Islands or Haida Gwaii consist of two main islands off the northwest coast of British Columbia. In addition to Graham Island and Moresby Island, there are approximately 150 smaller islands.

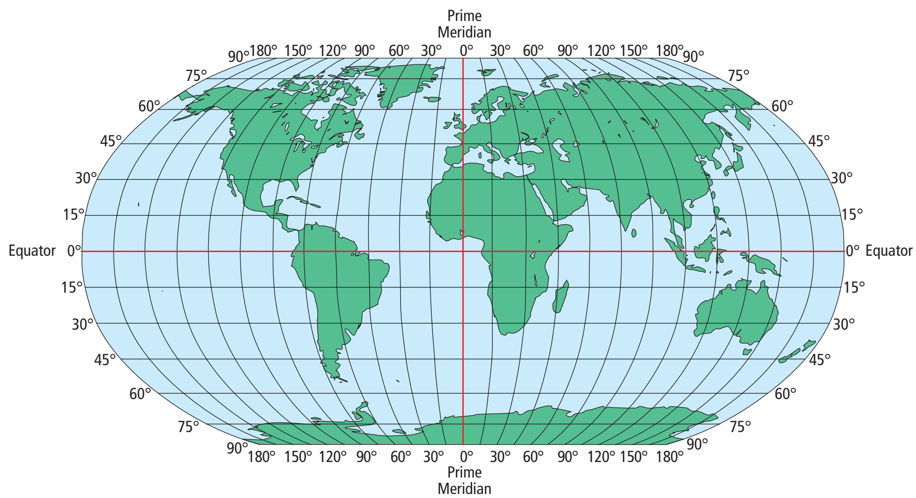
Solution

a) Extend the line that represents the course until it touches land, and then read the coordinates of the point. From the map, the coordinates where Anna will land are approximately 53.8° north latitude and −131.8° west longitude.

b) No, the line extended in the direction that Anna sailed from does not strike land. This is evidence that this extended line does not show the place where she started.



Literacy Link



Latitude and *longitude* is the most common grid system used for navigation.

- Each degree of latitude is approximately 111 km apart.
- Each degree of longitude varies from 0 to 111 km. A degree of longitude is widest at the equator and gradually decreases at the poles.

An example of a coordinate reading from a Canadian location might be 61° north latitude and −139° west longitude.

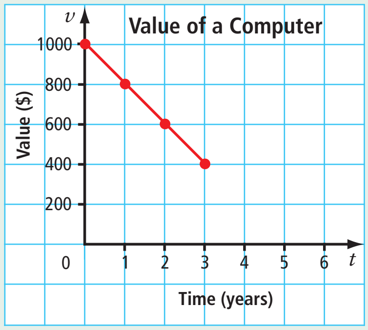
Show You Know

The value of a computer decreases over time. The graph shows the value of a computer after it was bought.

a) After what approximate period of time does the computer have no value?

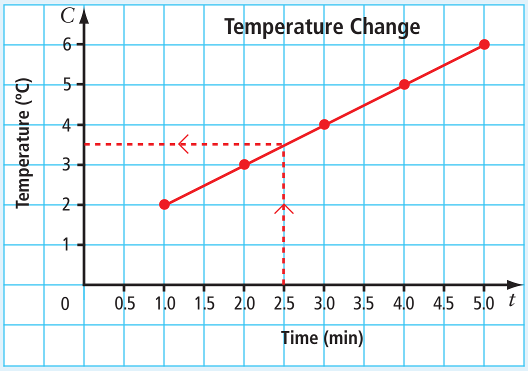
b) When was the computer worth approximately $200?

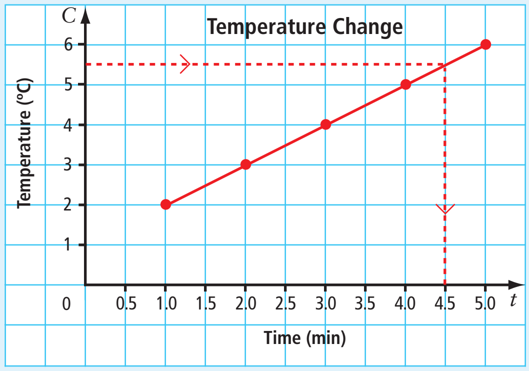
c) Is it appropriate to join the points with a straight line? Explain.



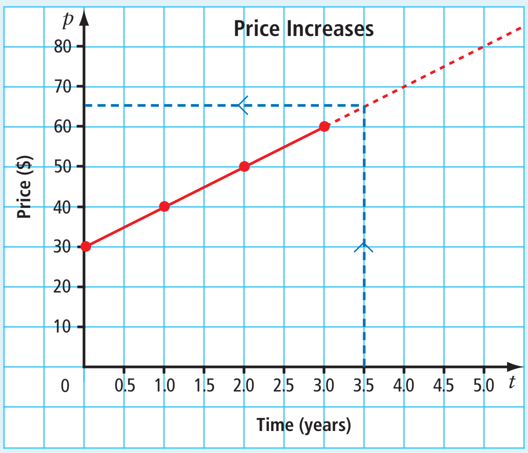
Key Ideas

- On a graph, you can use a line to interpolate values between known values.
  - Start with a known value for x.
  - Start with a known value for y.





- On a graph, you can extend a line to extrapolate values beyond known values.
  - Use a dashed line to extend the line beyond the known x-value or y-value.
  - Start with a known value for x.
  - Start with a known value for y.



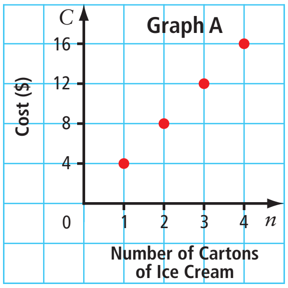
- Interpolation and extrapolation should be used only when it is reasonable to have values between or beyond the values on a graph.

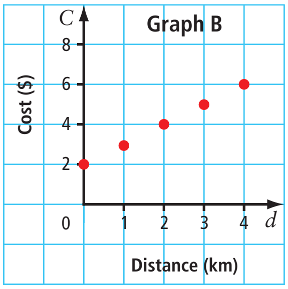
Check Your Understanding

Communicate the Ideas

1. Josh asked you to help him understand interpolation and extrapolation. Use an example and a graph to help explain how interpolation and extrapolation are similar and how they are different.

2. Grace says it would be reasonable to interpolate values on these graphs. Is she correct? Explain.



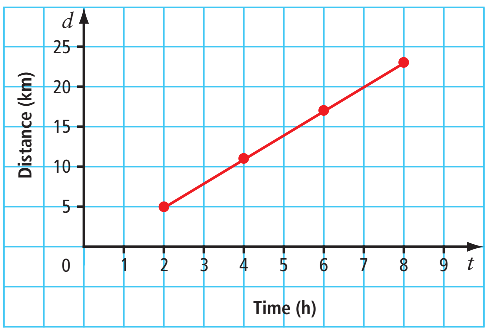


3. Develop a situation that involves a linear relation. Draw and label the corresponding graph. Develop a question and answer that requires extrapolating a value on the graph. Compare your work with that of a classmate.

Practise

For help with #4 to #7, refer to Example 1 on page 222.

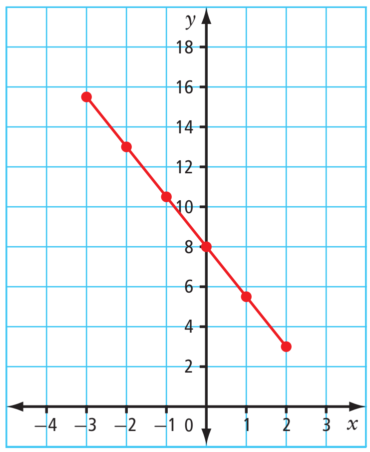
4. The graph shows a linear relation between distance and time.



a) What is the approximate value of the d-coordinate when $t = 5$? Explain the method you used to determine the answer.

b) What is the approximate value of the t-coordinate when $d = 20$?

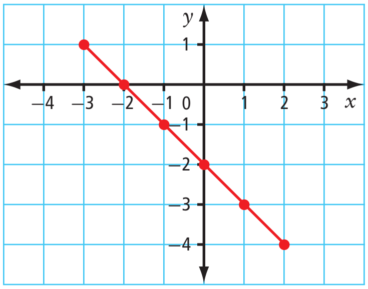
5. The graph shows a linear relation.



a) What is the approximate value of the y-coordinate when $x = -2.5$?

b) What is the approximate value of the x-coordinate when $y = 4$?

6. The graph shows a linear relation.



a) What is the approximate value of the y-coordinate when $x = 1.5$?

b) What is the approximate value of the x-coordinate when $y = 0.5$?

7. a) The table of values represents the distance that Sophie cycles in relation to time.

Time, *t* (h)	1	2	3	4	5	6
Distance, *d* (km)	12.5	25	37.5	50	62.5	75

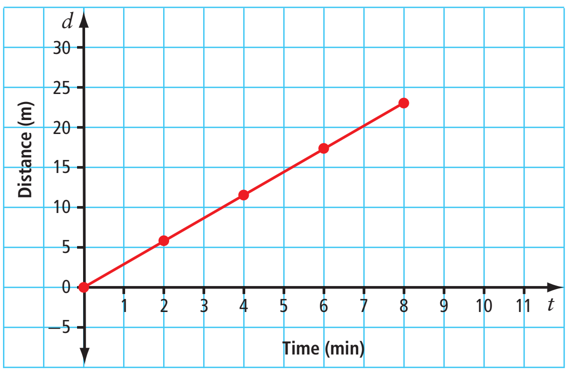
a) Plot the linear relation on a graph.

b) From the graph, determine the approximate distance Sophie has cycled after 2.5 h.

c) From the graph, approximate how long it takes Sophie to cycle 44 km.

For help with #8 to #11, refer to Example 2 on pages 223–224.

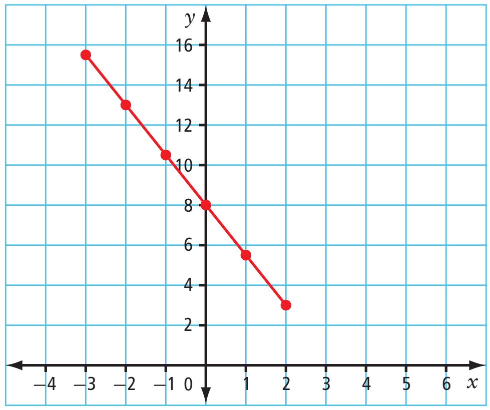
8. The graph shows a linear relation between distance and time.



a) What is the approximate value of the d-coordinate when $t = 10$?

b) What is the approximate value of the t-coordinate when $d = 33$?

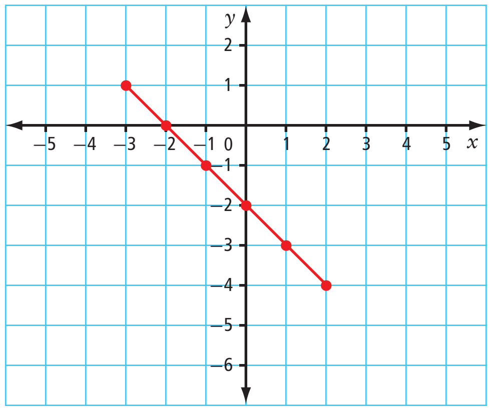
9. The graph shows a linear relation.



a) What is the y-coordinate when $x = -3$?

b) What is the x-coordinate when $y = 1$?

10. The graph shows a linear relation.



a) What is the approximate value of the y-coordinate when $x = -4$?

b) What is the approximate value of the x-coordinate when $y = -6$?

11. The table of values represents the drop in temperature after noon on a winter day.

Time, *t* (h)	1:00	2:00	3:00	4:00	5:00	6:00
Temperature, *d* (°C)	1	0	−1	−2	−3	−4

a) Plot the data on a graph.

b) From the graph, what is the approximate temperature at 6:30?

c) From the graph, determine the approximate time when the temperature is 2 °C.

Apply

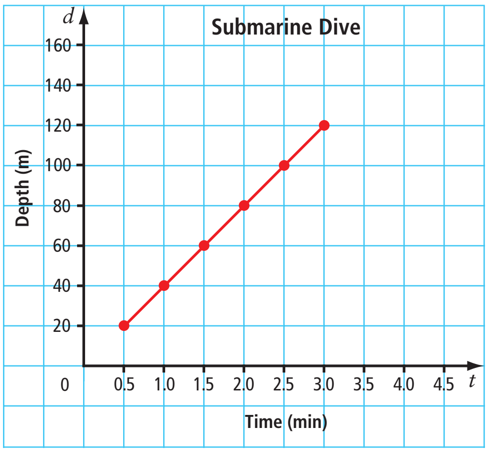
12. a) In a bulk food store, trail mix costs \$2.40 per 250 g. Plot the data on a graph.

Mass of Trail Mix, *m* (g)	250	500	750	1000	1250
Cost, *C* (\$)	2.40	4.80	7.20	9.60	12.00

b) From the graph, approximate how much 2000 g of trail mix would cost.

c) From the graph, approximate how much trail mix you would get for \$13.

13. The submarine HMCS *Victoria* can dive to a depth of 200 m.

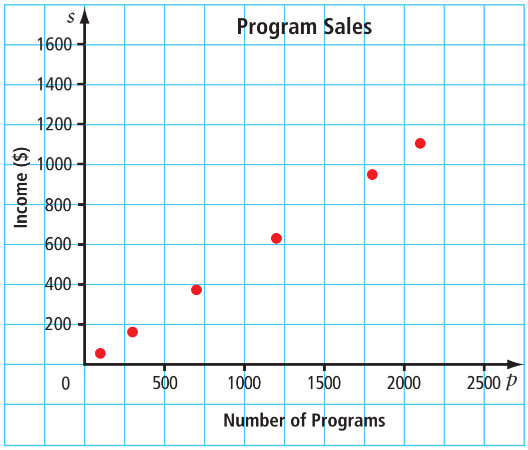


a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.

b) How long does it take *Victoria* to reach a depth of 140 m?

c) What is the submarine's depth after 4 min?

14. A grade 9 class earns a profit of 53¢ for each program they sell for the school play.

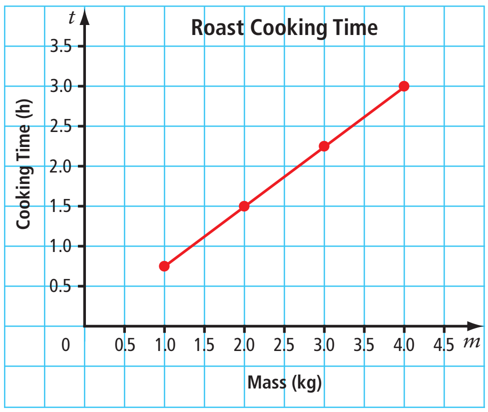


a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.

b) If 500 programs were sold, how much profit would the students make? What strategy did you use to solve the problem?

c) Approximately how many programs would students need to sell in order to earn \$2500?

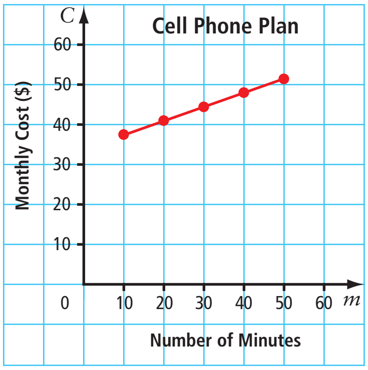
15. Sean learned in his cooking class that the time it takes to cook a roast depends upon its mass. The graph shows the relationship between cooking time and the mass of a roast.



From the graph, determine the approximate cooking time for a roast with each given mass.

a) 1.25 kg **b)** 2.25 kg **c)** 4.2 kg

16. A cell phone company charges a $33.95 monthly fee and long-distance charges at a rate of $0.35 per minute. The graph shows the monthly cost of phone calls based on the number of long-distance minutes.

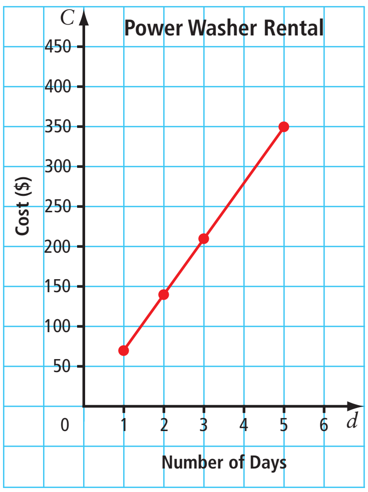


a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.

b) What would be the approximate monthly bill for 60 min of long-distance calls?

c) Approximately how many minutes of long-distance calls could you buy for $50?

17. The graph represents the relationship between the cost of renting a power washer and rental time.



a) How much does it cost to rent a power washer for four days? What is the cost per day? How do you know?

b) How long could you rent the power washer if you had $420?

Extend

18. The table shows the relationship between stopping distance and speed of a vehicle.

Speed, *s* (km/h)	15	30	45	60	75
Stopping Distance, *d* (m)	6	15	28	42	65

a) Plot the data on a graph. Draw a line to join the data points to best approximate the trend.

b) What happens to stopping distance as speed increases?

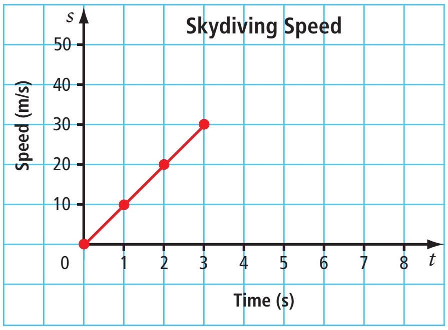
c) Estimate the stopping distances for speeds of 5 km/h, 55 km/h, and 80 km/h.

d) Estimate the speed before a driver applied the brake for stopping distances of 10 m, 50 m, and 100 m.

e) About how much farther is the stopping distance at 50 km/h than it is at 30 km/h? at 70 km/h than at 50 km/h?

f) Why do you think the graph is not a straight line?

19. The speed of a falling skydiver is shown for the first 3 s.



a) Approximately how long would it take for the skydiver to reach terminal velocity?

b) Approximately how far would the skydiver fall in that time?

c) Why do you think the graph is approximately a straight line?

Did You Know?

Terminal velocity is the maximum speed that a skydiver can reach when falling. Air resistance prevents most skydivers in free fall from falling any faster than 54 m/s.

Math Link

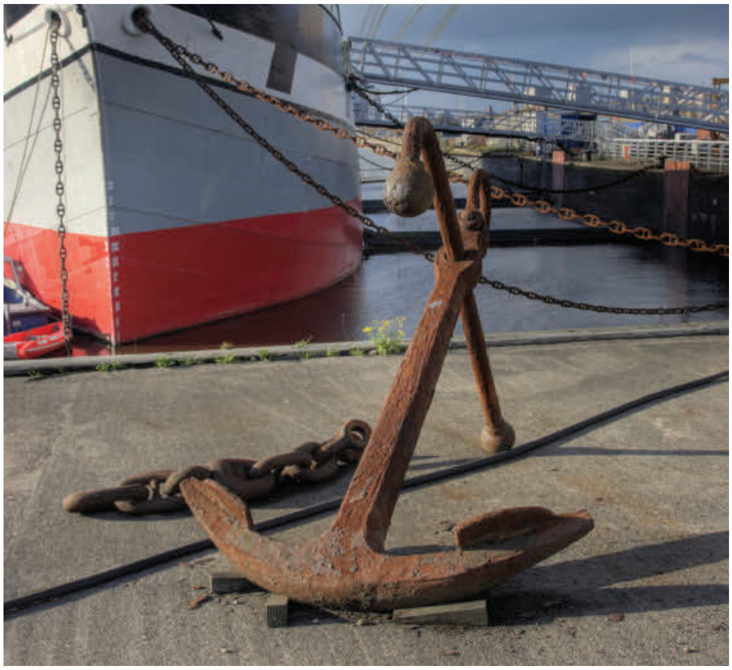
The area of the ocean called the Intertropical Convergence Zone (ITCZ) has little or no wind. Before propellers and motors, sailors used a relatively light anchor called a kedge to help them move across this region. The kedge anchor, which was attached to a line, was rowed out approximately 650 m ahead of the ship and dropped to the sea floor. A crew on the ship then grabbed the line and hauled it in to pull the ship to the anchor, a distance of 650 m. This process, called kedging, was repeated until the boat passed through the ITCZ.

a) Create a table of values showing the relationship between the number of kedges and the total distance travelled.

b) Plot the data on a graph. Label the graph.

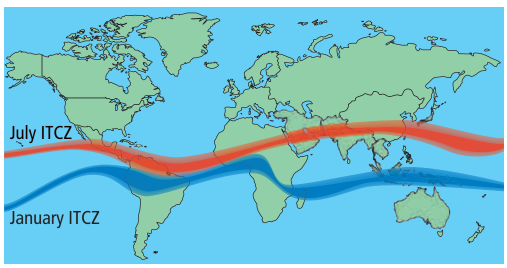
c) Determine the value for how many kedges it would take to traverse the width of the ITCZ.

d) How did the skills you have learned in this chapter help you solve part c)?



Did You Know?

The ITCZ is located between 5° north and 5° south of the equator and is approximately 1100 km wide. Note how the position of the ITCZ moves during the year.



WWW Web Link

To learn more about the Intertropical Convergence Zone (ITCZ), go to www.mathlinks9.ca and follow the links.

6.3 Graphing Linear Relations

Focus on...

After this lesson, you will be able to...
- graph linear relations
- match equations of linear relations with graphs
- solve problems by graphing a linear relation and analysing the graph

Tina is in charge of ordering water supplies for a cruise ship. She knows the amount of water required per day for each passenger and crew member as well as the amount of water reserves that the ship carries. She decides to use her knowledge of linear relations to draw a graph representing the relationship between the amount of water needed and the length of a cruise.

If Tina were to develop an equation, how could she determine if the graph and the equation represent the same relationship?

Explore Graphs of Linear Relations

Materials
- grid paper
- ruler

On a cruise, the average person requires a minimum of 4 L of water per day. The cruise ship has capacity for 1500 passengers and crew. The ship also carries a reserve of 50 000 L of water in case of emergency.

1. a) Use a method of your choice to determine how much water will be needed each day of a seven-day cruise.

b) On grid paper, plot the data and label your graph. Compare your graph with that of a classmate.

What values will you plot along the horizontal axis? along the vertical axis?

2. a) Predict how much water is needed for a ten-day cruise.

b) What linear equation represents the litres of water needed per day?

c) How could you verify your answer for part a)? Try out your strategy.

Reflect and Check

3. Do your graph and the equation represent the same relationship? Explain.

4. Discuss with a partner if it would be appropriate to interpolate or extrapolate values using a fraction of a day. Explain why or why not.

5. a) If the cruise ship used 152 000 L of water, approximately how long did the trip last? Compare the method you used with a classmate's.

b) Is there more than one way to answer part a)? Explain. Which method seems more efficient?

Link the Ideas

Example 1: Graph a Linear Equation

The world's largest cruise ship, *Freedom of the Seas*, uses fuel at a rate of 12 800 kg/h. The fuel consumption, f, in kilograms, can be modelled using the equation $f = 12\ 800t$, where t is the number of hours travelled.

a) Create a graph to represent the linear relation for the first 7 h.

b) Approximately how much fuel is used in 11 h? Verify your solution.

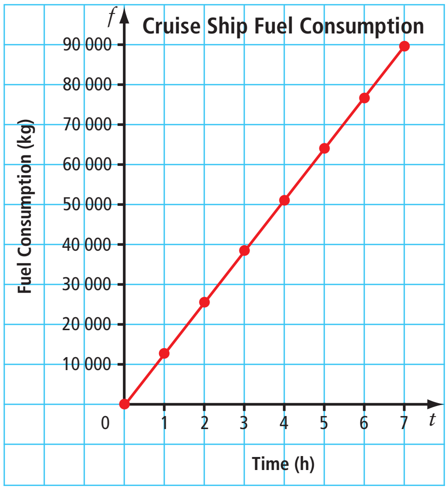
c) How long can the ship travel if it has approximately 122 000 kg of fuel? Verify your solution.

Solution

Method 1: Use Paper and Pencil

a) Create a table of values. Graph the coordinate pairs.

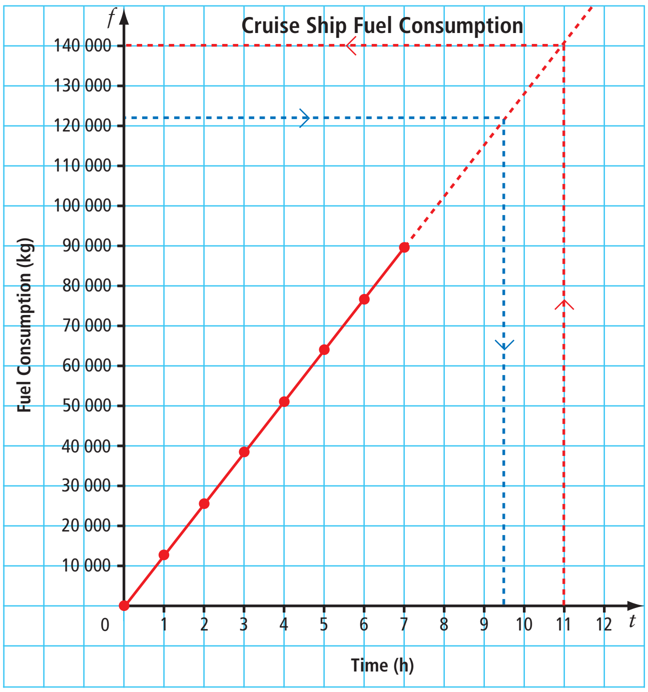
Time, t (h)	Fuel Consumption, f (kg)
0	0
1	12 800
2	25 600
3	38 400
4	51 200
5	64 000
6	76 800
7	89 600



Describe the connection between the equation and the graph.

b) Draw a straight line to connect the data points. Extend the line past the last data point.

Approximately 140 000 kg of fuel are used in 11 h.



What different methods might you use to represent and then solve the problem?

Check:
Substitute the value $t = 11$ into the equation $f = 12\ 800t$.

$f = 12\ 800(11)$
$= 140\ 800$

The approximate solution is correct.

c) The fuel will last approximately 9.5 h.

Check:
Substitute $f = 122\ 000$ into the equation and solve for t.

$122\ 000 = 12\ 800t$

$\frac{122\ 000}{12\ 800} = t$

$t \approx 9.53$

The approximate solution is correct.

Method 2: Use a Spreadsheet

a) In the spreadsheet, cell A1 has been labelled Time, t. Cell B1 has been labelled Fuel Consumption, f.

Enter the first eight values for t in cells A2 to A9. Then, enter the formula for the equation into cell B2. Use an = sign in the formula and * for multiplication. The value for t comes from cell A2.

	A	B
1	Time, t(h)	Fuel Consumption, f(kg)
2	0	=12800*A2
3	1	12800
4	2	25600
5	3	38400
6	4	51200
7	5	64000
8	6	76800
9	7	89600

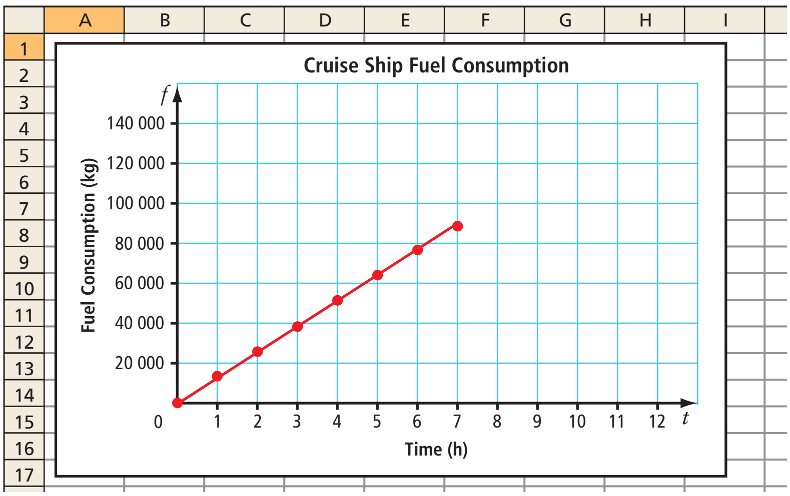
Tech Link

You could use a graphing calculator to graph this linear relation. To learn about how to do this go to www.mathlinks9.ca and follow the links.

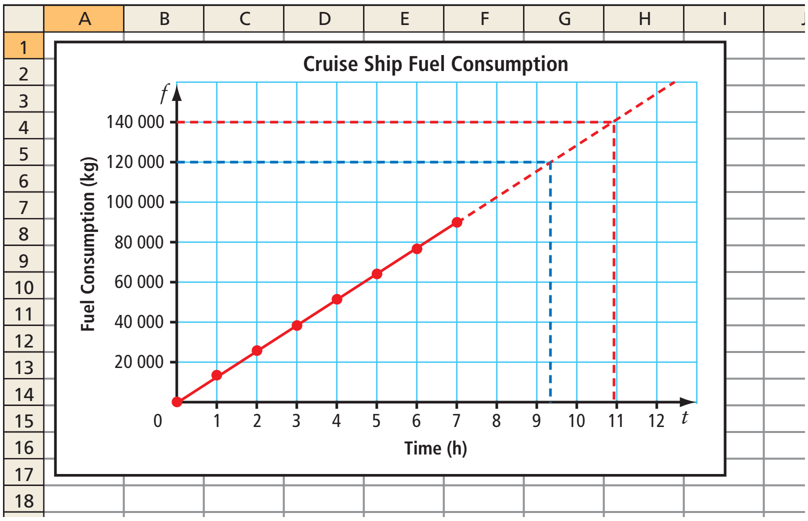
Use the cursor to select cells B2 down to B9.

Then, use the **Fill Down** command to enter the formula in these cells. The appropriate cell for t will automatically be inserted. For example, =12800*A6 will be inserted into cell B6.

Use the spreadsheet's graphing command to graph the table of values. Note that different spreadsheets have different graphing commands. Use your spreadsheet's instructions to find the correct command.



b) and **c)** From the menu, select **Add Trendline** to draw a straight line from the first data point to the last one. Extend the line past the last data point.



For part b), approximately 140 000 kg of fuel are used in 11 h. For part c), the fuel will last approximately 9.5 h.

Did You Know?

Fish finders operate using sonar, which uses sound waves to "see" objects underwater. The fish finder produces a sound wave and sends it through the water. When the sound wave meets an object within its range, it bounces back to the fish finder. The fish finder determines the depth of the object by measuring the time between when the sound wave was sent and when it returns. The fish finder then sketches the object on the screen.

Show You Know

a) Graph the linear relation $y = 2x - 5$.

b) Use the graph to estimate the value of y if $x = 8$.

c) Use the graph to estimate the value of x if $y = -4$.

Example 2: Determine a Linear Equation From a Graph

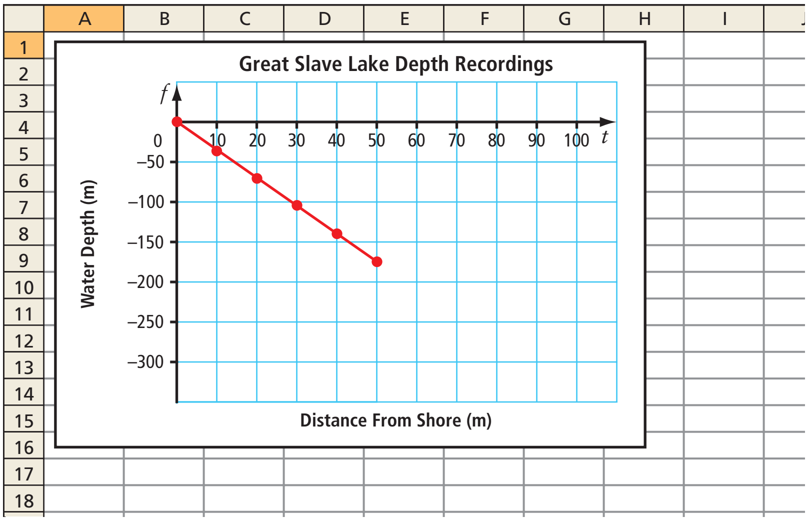
Great Slave Lake, which is located in the Northwest Territories, is the deepest lake in North America. It has a maximum depth of 614 m. Sam decided to check the depth using his fish finder. He collected the following data up to a depth of 180 m, which was the maximum depth that his fish finder could read.

Distance From Shore, d (m)	Water Depth, w (m)
0	0
10	−35
20	−70
30	−105
40	−140
50	−175

Literacy Link

A depth, such as 35 m, is expressed in different ways. In a table and a graph, use the negative value, −35. In a sentence, say "35 m below surface."

Sam used a spreadsheet to graph the data.



a) What linear equation does this graph represent? How do you know the equation matches the graph?

b) If this pattern continues, how far from shore would Sam be when the water is 614 m deep?

c) At what rate is the depth of the water decreasing?

d) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

Solution

a) Add a column to the table to help determine the pattern.

Distance From Shore, *d* (m)	Water Depth, *w* (m)	Pattern: Multiply *d* by −3.5
0	0	0
10	−35	−35
20	−70	−70
30	−105	−105
40	−140	−140
50	−175	−175

The water depth, w, decreases by 3.5 m for each 1-m increase in the distance from shore, d. The equation is $w = -3.5d$.

Check by substituting a known coordinate pair, such as (30, 105), into the equation.

Left Side $= -105$ Right Side $= -3.5(30)$

$= -105$

Left Side = Right Side

The equation is correct.

What is the connection between the graph and the equation?

Strategies

Solve an Equation

b) Substitute $w = 614$ into the equation and solve for d.

$$-614 = -3.5d$$
$$\frac{-614}{-3.5} = d$$
$$d \approx 175.4$$

Sam would be approximately 175.4 m from shore when the water is 614 m deep.

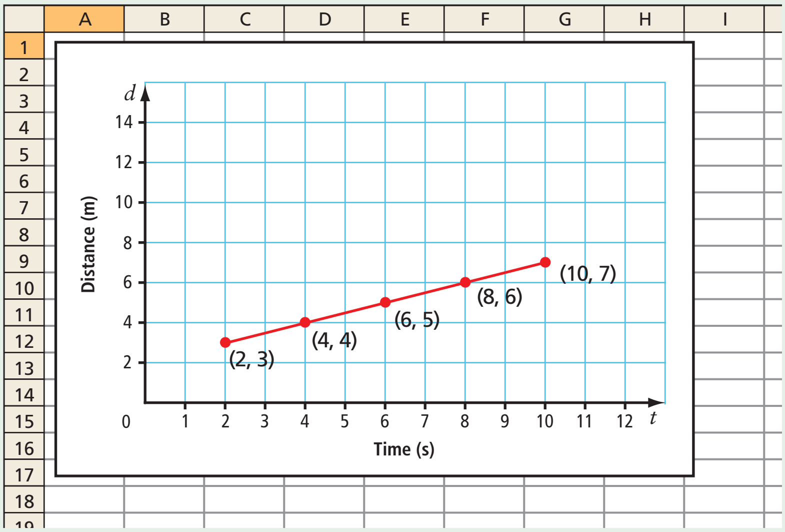
How else could you solve this problem?

c) The depth is decreasing at a rate of 3.5 m for each metre away from shore. The rate at which the water depth is decreasing is the coefficient of d in the equation.

d) Yes, it is reasonable to interpolate or extrapolate values between and beyond the given data points since the values for distance and depth exist. However, it is unreasonable to extrapolate values beyond the maximum depth of 614 m.

Show You Know

Identify the linear equation that represents the graph.



Example 3: Graph Horizontal and Vertical lines

For each table of values, answer the following questions:

Table 1

Time, t (s)	Distance, d (m)
0	6
30	6
60	6
90	6
120	6

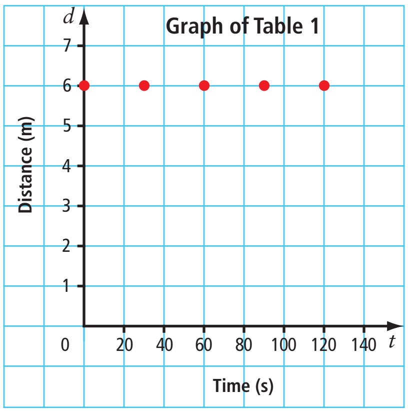
Table 2

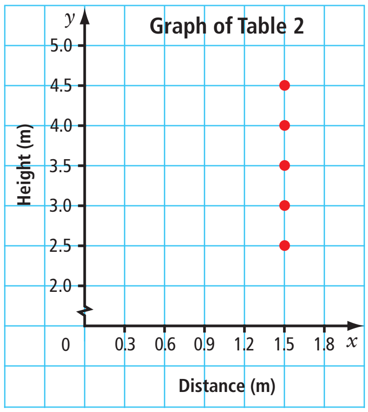
Distance, x (m)	Height, y (m)
1.5	2.5
1.5	3.0
1.5	3.5
1.5	4.0
1.5	4.5

a) Draw a graph to represent the table of values.
b) Describe a situation that the graph might represent.
c) Write the equation. Explain how you know the graph represents the equation.

Solution

a)





b) Table 1: The graph could show the relationship between distance and time when a pedestrian is waiting for a traffic light to change. The distance from the pedestrian to the opposite side of the road is constant.

Table 2: The graph could show the relationship between the height of a ladder and its distance from the wall where it is placed. The distance of the base of the ladder from the wall is constant as the ladder is extended.

Think of a different situation to represent each graph.

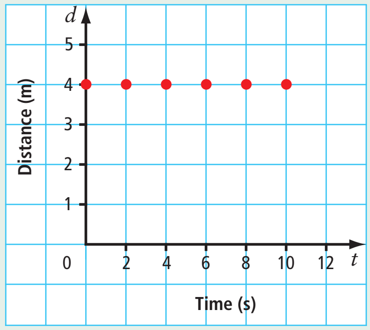
c) Table 1: The distance, d, remains constant for each interval of time. The equation is $d = 6$.
For each value of t in the table and the graph, the value of d is 6.

Table 2: The distance, x, remains constant for each interval of height. The equation is $x = 1.5$.
For each value of y in the table and the graph, the value of x is 1.5.

Show You Know

a) Write the linear equation that represents the graph.

b) Explain how you know the graph matches the equation.

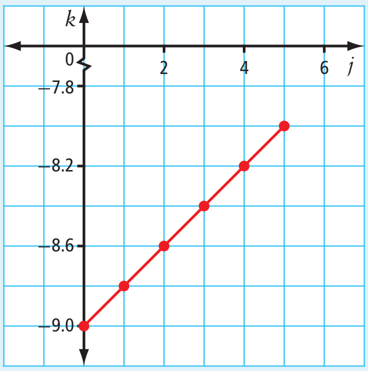


Key Ideas

- You can graph a linear relation represented by an equation.
  - Use the equation to make a table of values.
  - Graph using the coordinate pairs in the table. The graph of a linear relation forms a straight line.

$k = \frac{j}{5} - 9$

j	k
0	−9.0
1	−8.8
2	−8.6
3	−8.4
4	−8.2
5	−8.0



  - The graph of a linear relation can form a horizontal or a vertical line.
- You can use graphs to solve problems by interpolating or extrapolating values.

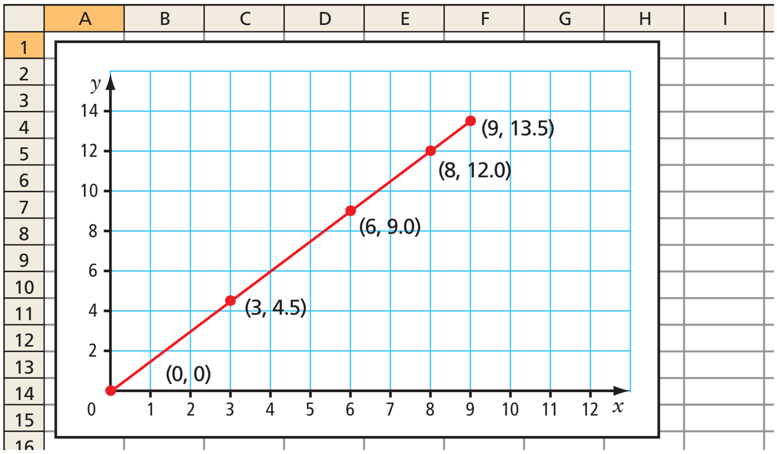
Check Your Understanding

Communicate the Ideas

1. You are given a linear equation. Describe the process you would follow to represent the equation on a graph. Use an example to support your answer.

2. Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.

3. a) Describe a real-life situation to represent the data on this graph.



b) Explain how you would determine the equation that represents the graph. Give your explanation to a classmate.

c) Can you interpolate or extrapolate values on this graph? Explain your thinking.

WWW Web Link

For practice matching graphs and linear equations, go to www.mathlinks9.ca and follow the links.

Practise

For help with #4 to #7, refer to Example 1 on pages 232–234.

4. Ian works part-time at a movie theatre. He earns \$8.25/h. The relationship between his pay, p, and the time he works, t, can be modelled with the equation $p = 8.25t$.

a) Show the relationship on a graph.

b) Explain how you know the graph represents the equation.

c) Ian works 8 h in one week. Use two methods to determine his pay.

5. Andrea is travelling by bus at an average speed of 85 km/h. The equation relating distance, d, and time, t, is $d = 85t$.

a) Show the relationship on a graph.

b) How long does it take Andrea to travel 300 km?

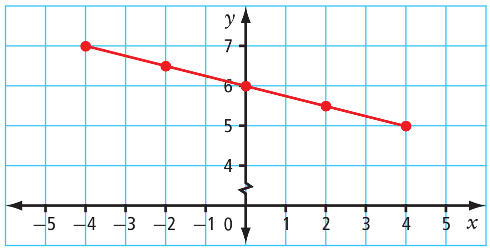
6. Choose the letter representing the graph that matches each linear equation.

a) $y = 5x$

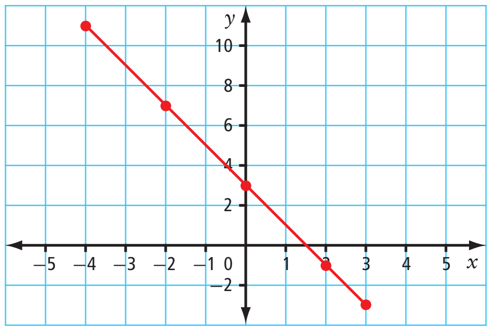
b) $y = -2x + 3$

c) $y = -\dfrac{x}{4} + 6$

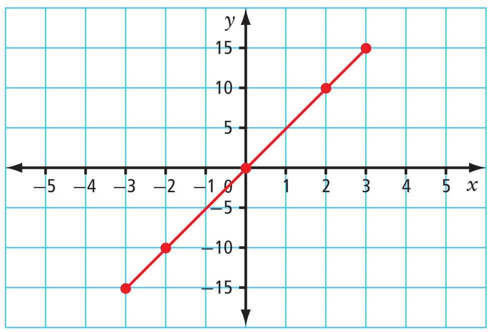
A



B



C



7. Create a table of values and a graph for each linear equation.

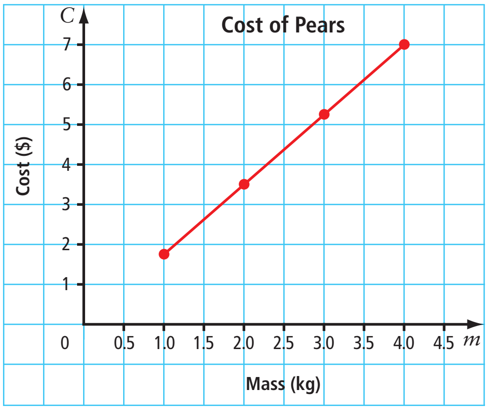
a) $x = 4$

b) $r = -3s + 4.5$

c) $m = \frac{k}{5} + 13$

For help with #8 to #11, refer to Example 2 on pages 234–236.

8. The graph shows the relationship between the cost, C, in dollars and the mass, m, in kilograms of pears.

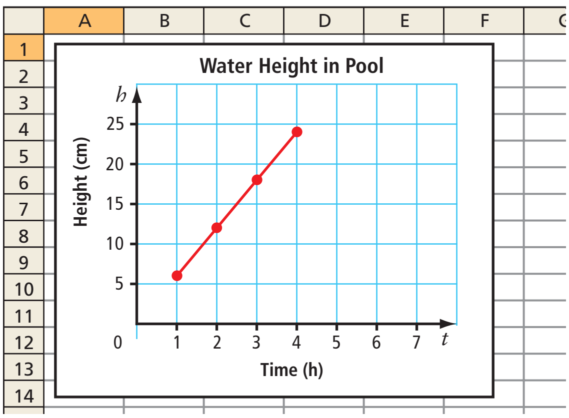


a) What is the linear equation?

b) How much could you buy for $5?

c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

9. The graph represents the relationship between the height of water in a child's pool, h, and the time, t, in hours as the pool fills.



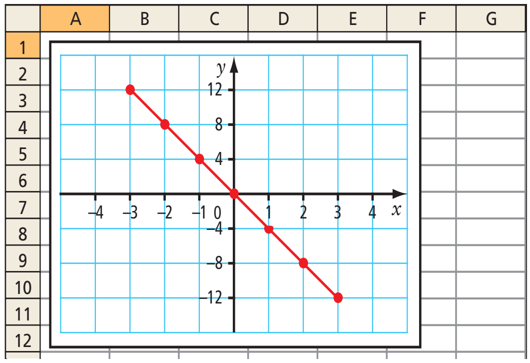
a) Determine the linear equation.

b) What is the height of the water after 5 h?

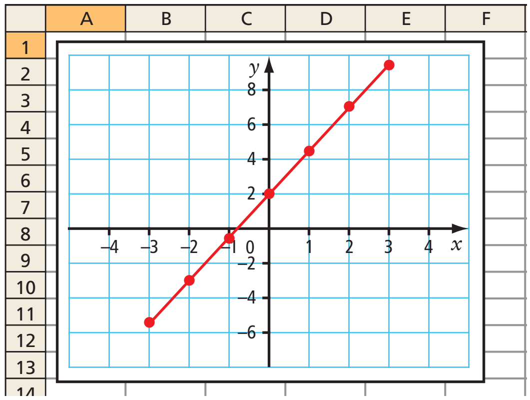
c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

10. Determine the linear equation that models each graph.

a)

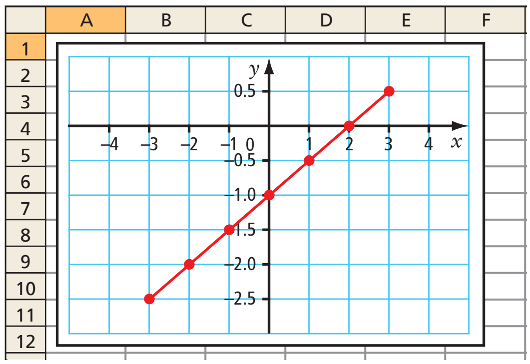


b)

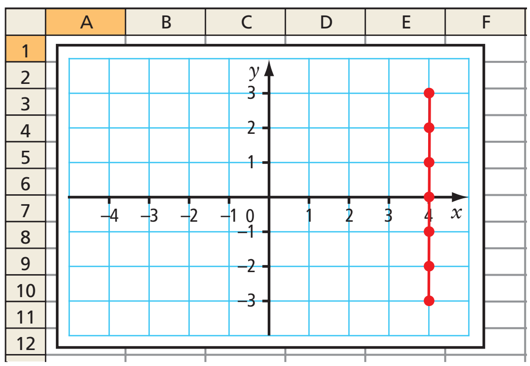


11. What linear equation does each graph represent?

a)



b)



12. Create a graph and a linear equation to represent each table of values.

a)

x	*y*
−3	−10
−2	−7
−1	−4
0	−1
1	2
2	5
3	8

b)

r	*t*
−3	−2.5
−2	−1.0
−1	0.5
0	2.0
1	3.5
2	5.0
3	6.5

c)

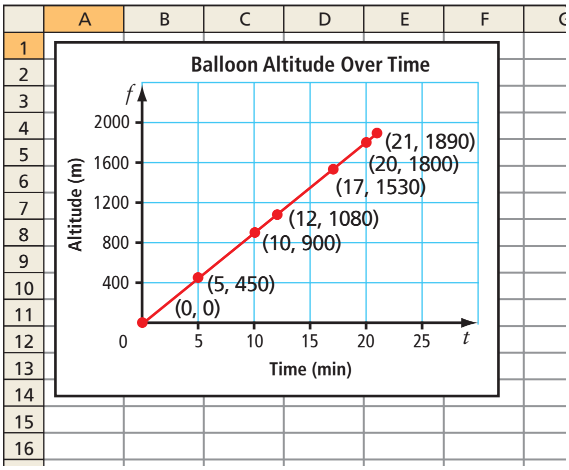
f	*z*
−3	−3
−2	−3
−1	−3
0	−3
1	−3
2	−3
3	−3

d)

h	*n*
−3	−0.75
−2	−0.5
−1	−0.25
0	0
1	0.25
2	0.5
3	0.75

Apply

13. The graph represents the altitude of a hot-air balloon the first 20 min after it was released.



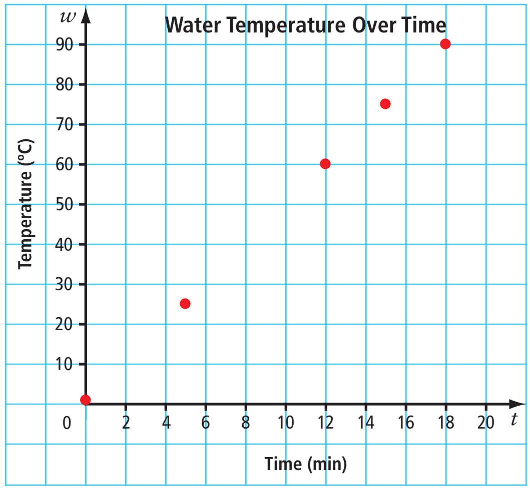
a) What was the approximate altitude of the balloon after 15 min?

b) Estimate how long it took for the balloon to rise to an altitude of 1 km.

c) What linear equation models the graph?

d) How fast is the balloon rising?

14. Sanjay conducted an experiment to determine how long it takes to heat water from 1 °C to its boiling point at 100 °C. He plotted his data on a graph.



a) Approximately how long did it take for the water to reach boiling point? Explain your reasoning.

b) What was the temperature of the water after 10 min?

c) At what rate did the water temperature increase? Explain your reasoning.

15. Paul drives from Edmonton to Calgary. He uses a table to record the data.

Time, *t* (h)	Distance, *d* (km)
0.5	55.0
0.9	99.0
1.2	132.0
1.5	165.0
2.3	253.0
2.7	297.0

a) Graph the linear relation.

b) How far did Paul drive in the first 2 h?

c) How long did it take Paul to drive 200 km?

d) Write the equation that relates time and distance.

e) What was Paul's average driving speed? What assumptions did you make?

WWW Web Link

To learn about using a graphing calculator to enter data on a table and plot the data on a graph, go to www.mathlinks9.ca and follow the links.

16. The relationship between degrees Celsius (°C) and degrees Fahrenheit (°F) is modelled by the equation $F = \frac{9}{5}C + 32$.

a) Graph the relationship for values between −50 °C and 120 °C.

b) Water boils at 100 °C. What is this temperature in degrees Fahrenheit?

c) Water freezes at 0 °C. How did you represent this on your graph?

d) At what temperature are the values for °C and °F the same?

17. Scuba divers experience an increase in pressure as they descend. The relationship between pressure and depth can be modelled with the equation $P = 10.13d + 102.4$, where P is the pressure, in kilopascals, and d is the depth below the water surface, in metres.

a) Graph the relationship for the first 50 m of diving depth.

b) What is the approximate pressure at a depth of 15 m? Verify your answer.

c) The maximum pressure a scuba diver should experience is about 500 kPa. At what depth does this occur? Verify your answer.

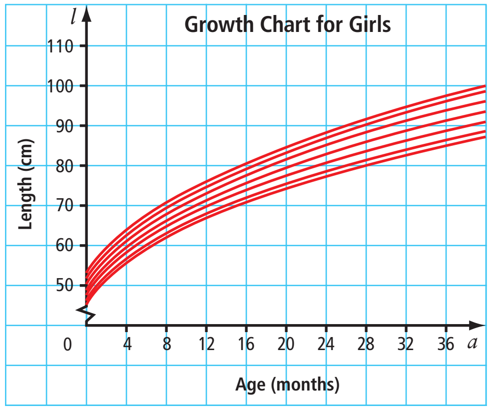
d) What does "+ 102.4" represent in the equation? How is it represented on the graph?

Did You Know?

After deep or long dives, scuba divers need to undergo decompression. They do this by ascending to the surface slowly in order to avoid decompression sickness, also known as the bends.

Extend

18. The graph shows the normal range of length for girls from birth to age 36 months.



a) For what age range does girls' growth appear to represent a linear relation?

b) For what age range, does girls' growth appear to represent a non-linear relation?

19. Janice left the school at 12 noon riding her bike at 20 km/h. Flora left school at 12:30 riding her bike at 24 km/h.

a) Draw a distance–time graph to plot the data for both cyclists during the first four hours. Use a different colour for each cyclist.

b) How can you tell from the graph that Flora has caught up to Janice?

c) About what time did Flora catch up to Janice?

d) If Janice and Flora continued to ride at their respective speeds, at what time would they again be apart by a distance of 2 km?

20. An online music download site offers two monthly plans. Plan A offers \$10 plus \$1 per download and Plan B offers \$1.50 per download.

a) Graph both linear relations on the same grid.

b) Explain the conditions under which each deal is better.

21. Simple interest is paid according to the formula $I = p \times r \times t$, where p is the principal, r is the rate of interest per year, and t is the time in years. The interest is not added to the principal until the end of the time period. Canada Savings Bonds offer a simple interest bond payable at 3.5% per year up to a maximum of ten years.

a) Create a table of values to show the interest earned on a \$1000 bond for the ten-year period.

b) Use a graph to show the interest earned over ten years.

c) How many years would it take to earn \$100 interest? \$200 interest?

d) If you could leave the principal beyond the ten-year period, estimate the number of years it would take to earn \$500 interest.

Math Link

The world's fastest submarines can reach speeds of 74 km/h in 60 s, starting from rest. If a submarine is already moving, then the time to reach its top speed will differ.

a) Choose four different starting speeds up to a maximum of 74 km/h. For each speed, assume that the acceleration is the same. For each speed include:
- a table of values
- a linear equation and a graph to represent the relationship between speed and time

b) Describe each graph. Identify any similarities and differences you observe between the graphs and the equations.

Did You Know?

A student team from the University of Québec set a new world speed record for the fastest one-person, non-propeller submarine. In 2007, the submarine, *OMER 6*, reached a speed of 4.642 knots (8.6 km/h) in the International Submarine Races.

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Chapter 6 Review

Key Words

For #1 to #5, unscramble the letters for each term. Use the clues to help you.

1. R A N E I L R A I N E T L O
a pattern made by a set of points that lie in a straight line when graphed

2. P L E X A T R O T E A
estimate values beyond known data

3. T S T O N C A N
in $y = 4x + 3$, the number 3 is an example

4. E L I N A R Q U E I O N A T
an equation that relates two variables in such a way that the pattern forms a straight line when graphed

5. T R I P O L E N E A T
estimate values between known data

6.1 Representing Patterns, pages 210–219

6. a) Make a table of values for the toothpick pattern.

Figure 1 Figure 2 Figure 3

b) Describe the pattern.

c) Develop an equation relating the number of toothpicks to the figure number.

d) How many toothpicks are in Figure 10? Verify your answer.

e) How do the numerical values in the equation represent the pattern?

7. Derek has \$56 in his bank account. He plans to deposit \$15 every week for a year.

a) Create a table of values for his first five deposits.

b) What equation models this situation?

c) How much money will Derek have in his account after 35 weeks?

d) How long will it take him to save \$500?

8. Taylor works at a shoe store. She makes \$50 per day plus \$2 for every pair of shoes she sells.

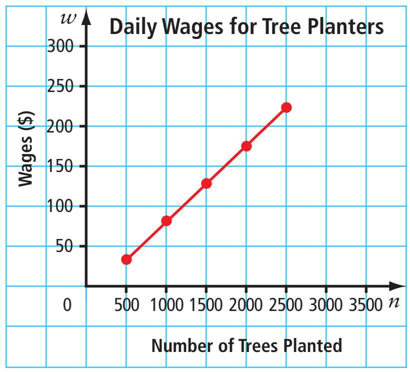
a) Create a table of values to show how much she would earn for selling up to ten pairs of shoes in one day.

b) Develop an equation to model this situation.

c) How much money will Taylor make in a day if she sells 12 pairs of shoes? Use two methods for solving the problem.

6.2 Interpreting Graphs, pages 220–230

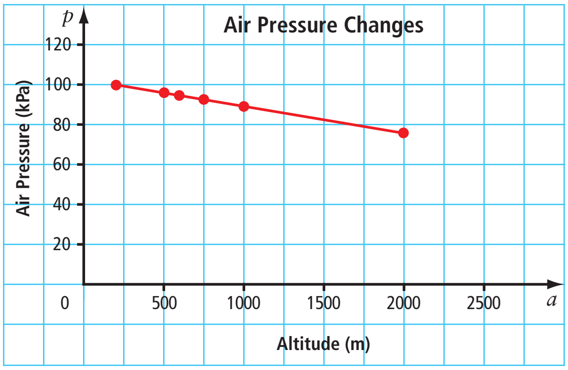
9. Many tree planters are paid according to how many trees they plant. The following graph shows the daily wages earned at a rate of \$0.09 per tree planted.



a) Approximately how much would a tree planter who planted 750 trees earn in one day?

b) In order to earn \$250 in one day, approximately how many trees would a planter need to plant?

10. The graph shows the relationship between air pressure, in kilopascals, and altitude, in metres.



a) What is the approximate air pressure at an altitude of 1500 m? 2400 m?

b) Approximately at what altitude is the air pressure 90 kPa? 60 kPa?

c) Does it make sense to interpolate or extrapolate values on this graph? Explain.

11. There are 15 schools in an urban school district. The table shows data about the student and teacher populations for eight of the schools.

Students	100	250	300	450	700	150	1025	650
Teachers	9	15	17	23	33	11	46	31

a) Graph the relationship between the number of students and teachers.

b) How many teachers might be in a school that has 850 students? 1200 students?

c) How many students might attend a school that employs 30 teachers? 50 teachers?

6.3 Graphing Linear Relations, pages 231–243

12. The cost of renting a snowboard can be calculated using the equation $C = 40 + 20d$, where C is the rental cost, in dollars, and d is the number of rental days.

a) Graph the linear relation for the first five days.

b) From the graph, what is the approximate cost of renting the snowboard for one day? seven days?

c) If buying a snowboard costs $300, use your graph to approximate how many days you could rent a board before it becomes cheaper to buy it.

d) Describe another method you could use to solve parts b) and c).

13. Graph the linear relation represented in the table of values.

Time (h)	Distance (km)
0.5	52.5
1.0	105.0
1.5	157.5
2.0	210.0
2.5	262.5
3.0	315.0
3.5	367.5
4.0	420.0

a) Describe a situation that might lead to these data.

b) Develop a linear equation to model the data.

c) What do the numerical coefficients and constants in the equation tell you?

14. A parking lot charges a flat rate of $3.00 and $1.75 for each hour or part of an hour of parking.

a) Create a table of values for the first 8 h of parking.

b) Graph the linear relation.

c) Use the graph to approximate how much it would cost to park for 4 h.

d) Using the graph, approximately how long could you park if you had $15.25?

e) What equation models this situation?

Chapter 6 Practice Test

For #1 to #3, select the best answer.

Use the pattern below to answer #1 and #2.

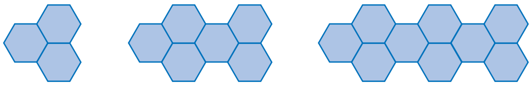


Figure 1 Figure 2 Figure 3

1. Which table of values best represents the pattern?

A

Figure Number (*f*)	1	2	3	4
Number of Sides (*s*)	18	36	54	72

B

Figure Number (*f*)	1	2	3	4
Number of Sides (*s*)	18	28	38	48

C

Figure Number (*f*)	1	2	3	4
Number of Sides (*s*)	12	20	28	36

D

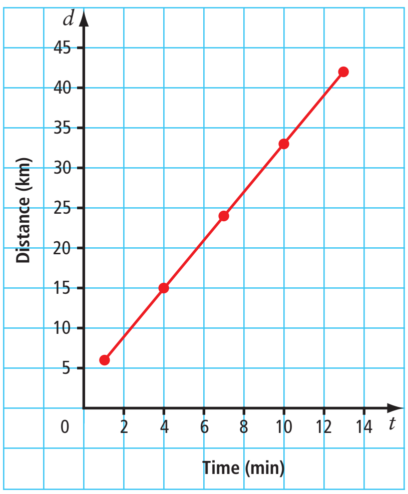
Figure Number (*f*)	1	2	3	4
Number of Sides (*s*)	12	24	36	48

2. Which equation represents the pattern?

A $s = 12f$ **B** $s = 8f + 4$

C $s = 10f + 8$ **D** $s = 18f$

3. Which equation represents this graph?



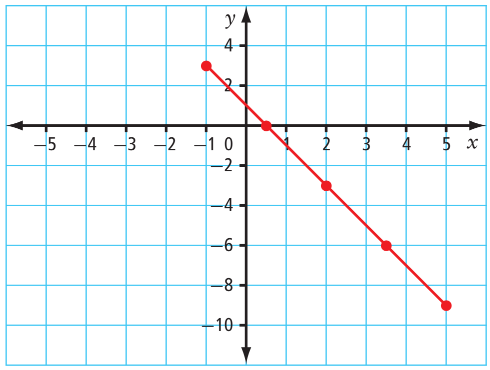
A $d = 2t + 4$ **B** $d = 4t - 1$

C $d = 3t + 3$ **D** $d = t + 5$

Complete the statements in #4 and #5.

4. When $x = 1.5$ on the graph, the approximate y-coordinate is ■.

5. When $y = -8$ on the graph, the approximate x-coordinate is ■.



Short Answer

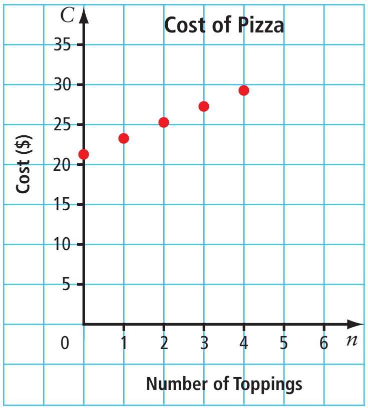
6. A number pattern starts with the number −2. Each number is 4 less than the previous number.

a) Make a table of values for the first five numbers in the pattern.

b) What equation can be used to determine each number in the pattern? Verify your answer.

c) What is the value of the 11th number in the pattern?

7. A cheese party pizza costs \$21.25. The graph shows the cost of adding additional toppings.



a) What is the approximate cost of a party pizza with five toppings?

b) Is it reasonable to interpolate values on this graph? Explain.

8. Create a table of values and a graph for each equation.

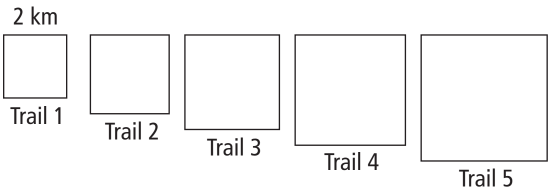
a) $y = -2x + 6$ **b)** $y = 2x - 6$

c) $y = 6$

9. How are the graphs in #8 similar? How are they different?

Extended Response

10. A cross-country ski park contains five different trails. The diagram shows the trails, with each trail being successively larger.



Each side length of the shortest trail is 2 km. The side length of each consecutive trail is 0.5 km longer than the previous one.

a) Construct a table of values to show the relationship between the trail number and the total distance of each trail.

b) What equation represents the relationship?

c) Graph the linear relation.

d) If a sixth trail were added, what would be its total distance?

Math Link: Wrap It Up!

You are planning a canoe trip with some friends. Where are you going? How long will your trip be? How many people are going?

You are in charge of ordering food supplies to meet the energy requirements of your group. For the trip, the amount of food energy required by a canoeist can be modelled by the equation $a = \frac{C}{100} - 17$, where a represents the person's age and C represents the number of calories.

Use the Internet, travel brochures, or other sources to find information about your trip.

a) Write a paragraph describing your trip.

b) Create a table of values for your data about total food energy requirements for the group.

c) Graph the linear relation.

d) Develop a problem based on your graph that also includes interpolation and extrapolation and provide a solution. Show your work.

Waterton Park, Alberta

Challenges

Hot-Air Ballooning

On January 15, 1991, the Pacific Flyer completed the longest flight ever made by a hot air balloon. The balloon flew 7671.91 km from Japan to northern Canada.

The balloon is designed to fly in the transoceanic jet streams. The Pacific Flyer hitched a ride on these strong winds and was swept high above the ocean.
The balloon reached a ground speed of 394 km/h. This is the fastest ground speed ever achieved by a hot-air balloon!

Thousands of Canadians enjoy a far less extreme ballooning experience each year. The following altitudes were recorded for two hot-air balloons at the indicated times.

	Time	Altitude	Time	Altitude
Hot-Air Balloon 1	8:00 a.m.	100 m	9:00 a.m.	5100 m
Hot-Air Balloon 2	8:15 a.m.	8100 m	8:45 a.m.	6600 m

Assume that each balloon is ascending or descending at a steady rate.

Justify your answers to each the following questions.

1. How far did Balloon 1 ascend between 8 a.m. and 9 a.m.? Based on your answer, calculate the speed of ascent in metres per hour. Show your calculations.

2. How far did Balloon 2 descend between 8:15 a.m. and 8:45 a.m.? Based on your answer, calculate the speed of descent in metres per hour. Show your calculations.

3. a) At what time will both balloons be at the same altitude?

b) What is the altitude?

4. What is the altitude of each balloon at 8:20 a.m.?

5. At what time would you expect Hot Air Balloon 1 to reach 8100 m?

WWW Web Link

For more information about hot-air balloon records, go to www.mathlinks9.ca and follow the links.

Opening a Fitness Club

You and a friend are planning to open a new fitness club. You have researched two clubs that offer the services you would like to provide. These clubs offer the following membership plans if you join for a year:

Workout Club
one month free,
then $35 per month

Get Fit Club
initial fee of $200,
then $25 per month

1. Develop a membership plan that will make the fees for your club competitive. The plan should attract members and generate more income than at least one of the other club plans.

a) What is your plan?

b) Explain how your plan will attract members.

c) Explain how your plan may result in earning more profit than the other clubs.

2. Suppose a potential member has $1000 to spend. Which club offers the best deal? Show your work.

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