CHAPTER

8

Solving Linear Equations

A balanced diet is one of the keys to good health, physical and mental development, and an active life. Making healthy food choices requires knowledge of your nutritional needs and of the nutrients found in foods. There are resources to help you. An example is Canada's Food Guide. It stresses the importance of eating a variety of food such as vegetables, fruits, and whole grains. It is also important to control your intake of fat, sugar, and salt.

Web Link

To get a copy of Canada's Food Guide, go to www.mathlinks9.ca and follow the links. The links also provide useful resources on how to use the food guide. These resources include "Eating Well with Canada's Food Guide" and "Eating Well with Canada's Food Guide—First Nations, Inuit and Métis."

What You Will Learn

- to model problems using linear equations
- to solve problems using linear equations

Key Words

equation
variable
numerical coefficient
constant
opposite operation
distributive property

Literacy Link

A concept map can help you visually organize your understanding of math concepts.

Create a concept map in your math journal or notebook. Make each oval large enough to write in. Leave enough space to attach additional ovals to each strategy shown. As you work through the chapter, complete the concept map.

For each strategy
- develop an example
- list the steps for solving the equation

Discuss your strategies with a classmate. You may wish to add to or correct what you have written.

with multiplication and division
with two operations
Solving Equations
with grouping symbols
with variables on both sides

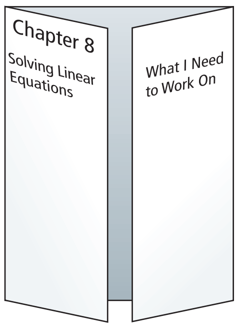
FOLDABLES™
Study Tool

Making the Foldable

Materials
- sheet of 11 × 17 paper
- ruler
- four sheets of 8.5 × 11 paper
- scissors
- stapler
- index cards

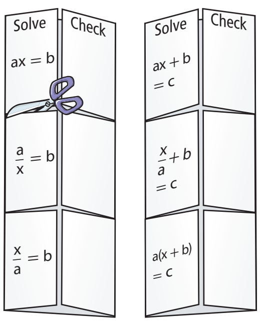
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges to meet at the midpoint. Label it as shown.



Step 2

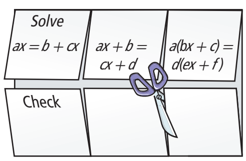
Fold the short side of a sheet of 8.5 × 11 paper in half. Pinch it at the midpoint. Fold the outer edges to meet at the midpoint. Fold in three the opposite way. Make four cuts as shown through one thickness of paper, forming six tabs.



Repeat Step 2 to make another shutter-fold booklet. Label them as shown.

Step 3

Fold the long side of a sheet of 8.5 × 11 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Fold in three the opposite way. Make four cuts through one thickness of paper, forming six tabs. Label the tabs as shown.

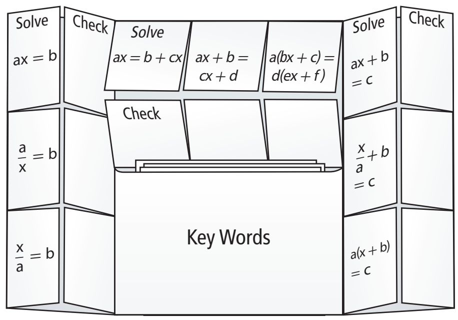


Step 4

Use half a sheet of 8.5 × 11 paper to create a pocket for storing Key Words and examples of linear equations.

Step 5

Staple the three booklets and the pocket you made into the Foldable from Step 1 as shown.



Using the Foldable

As you work through Chapter 8, use the shutter-fold booklets on the left and right panels to check your understanding of the concepts. Write an equation below the left tab, in the form indicated. Write the solution to the equation under the right tab. Use the centre shutterfold in a similar manner, writing the equation under the top tab and the solution under the bottom tab.

Use the back of the Foldable to record your ideas for the Math Link: Wrap It Up!

On the front of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

Math Link

Solve Problems Involving Nutrition

Some problems that involve nutrition can be modelled using linear equations. In this chapter, you will use linear equations of different forms to model problems involving various foods.

1. Model each problem with an equation. Then, solve the equation. Share your method with your classmates.

a) The mass of carbohydrate in a medium-sized peach is 5 g less than the mass of carbohydrate in a medium-sized orange. The peach contains 10 g of carbohydrate. What mass of carbohydrate is in the orange?

b) Half a pink grapefruit contains 47 mg of vitamin C. What mass of vitamin C does a whole pink grapefruit contain?

c) One litre of skim milk contains 1280 mg of calcium. What is the mass of calcium in one 250-mL serving of skim milk?

d) A 250-mL serving of baked beans in tomato sauce contains 11 g of fibre. This mass of fibre is 1 g more than the mass of fibre in two 85-g servings of whole wheat pasta. What mass of fibre is in one 85-g serving of whole wheat pasta?

e) The mass of potassium in a medium-sized apple, including the skin, is about 160 mg. This mass is 10 mg more than one third of the mass of potassium in an average-sized banana. What mass of potassium is in the banana?

f) One serving of a snack contains 250 mL of dried apricots and 125 g of low-fat, plain yogurt. Three servings of this snack contain 36 g of protein. If 125 g of yogurt provides 7 g of protein, how much protein is in 250 mL of dried apricots?

2. a) Develop two different problems involving nutrition that can be modelled using linear equations. Use the Internet or the library to research the nutritional information. Make sure you can solve the problems you create.

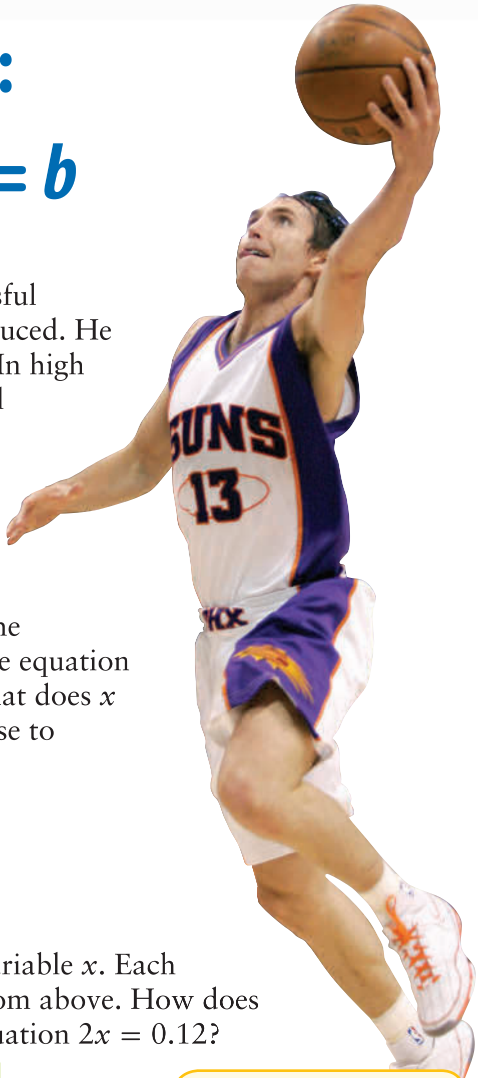
b) Ask a classmate to solve your problems. Verify your classmate's solutions.

WWW Web Link

To learn more about nutrition, healthy eating, and the nutritional values of different foods, go to www.mathlinks9.ca and follow the links

8.1

Solving Equations: $ax = b, \frac{x}{a} = b, \frac{a}{x} = b$



Focus on…

After this lesson, you will be able to…

- model problems with linear equations that can be solved using multiplication and division
- solve linear equations with rational numbers using multiplication and division

Steve Nash is arguably the most successful basketball player that Canada has produced. He grew up in Victoria, British Columbia. In high school, he led his team to the provincial championship and was the province's player of the year. He has since become one of the top players in the National Basketball Association (NBA).

One year, Steve scored 407 points for the Phoenix Suns in 20 playoff games. If the equation $20x = 407$ represents this situation, what does x represent? What operation could you use to determine the value of x?

Did You Know?

A Canadian, Dr. James Naismith, invented the game of basketball in 1891. At the time, he was teaching in Springfield, Massachusetts.

WWW Web Link

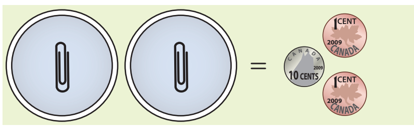
To learn more about Steve Nash's life, his career, and his work in communities, go to www.mathlinks9.ca and follow the links.

Materials

- coins
- paper cups or small containers
- paper clips

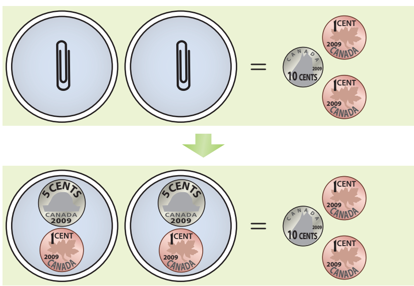
Explore Equations Involving Multiplication and Division

1. a) Each paper clip represents the variable x. Each circle represents a cup viewed from above. How does the diagram model the linear equation $2x = 0.12$?



What other combination of coins could you use to represent 0.12?

b) How does the diagram model the solution to the equation in part a)? What is the solution?



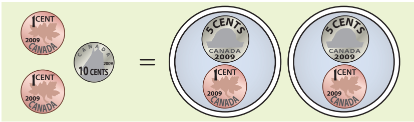
c) Explain how the second part of the diagram in part b) can also model the equation $0.06y = 0.12$. What is the solution? Explain.

2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $3x = 0.6$

b) $0.05y = 0.25$

3. a) How does reversing the second part of the diagram from #1b) model the solution to the equation $\frac{n}{2} = 0.06$?



b) What is the solution? Explain.

c) Describe how the diagram in part a) can also model the solution to the equation $\frac{0.12}{k} = 0.06$. What is the solution? Explain.

4. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $\frac{x}{3} = 0.05$

b) $\frac{0.33}{c} = 0.11$

Literacy Link

An *equation* is a statement that two mathematical expressions have the same value. Examples of equations include $3x = -2$, $\frac{y}{2} = 1$, and $z = -2.7$.

In the equation $1.2d + 3.5 = -1.6$,

- *d* is the *variable*, which represents an unknown number
- 1.2 is the *numerical coefficient*, which multiplies the variable
- 3.5 and −1.6 are *constants*

Reflect and Check

5. a) How can you model solutions to equations of the form $ax = b$, $\frac{x}{a} = b$, and $\frac{a}{x} = b$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

6. a) When a basketball player takes the ball away from an opposing player, it is called a *steal*. In his first nine seasons in the NBA, Steve Nash averaged 0.8 steals per game. Write and solve an equation that can be used to determine how many games it took, on average, for Steve to achieve four steals.

b) Use at least one other method to solve the problem. Share your solutions with your classmates.

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Link the Ideas

Example 1: Solve One-Step Equations With Fractions

Solve each equation.

a) $2x = \frac{3}{4}$ **b)** $\frac{m}{3} = -\frac{2}{5}$ **c)** $-2\frac{1}{2}k = -3\frac{1}{2}$

Solution

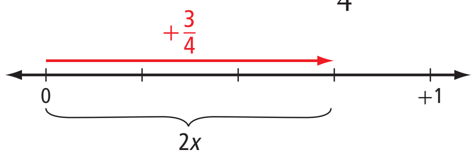
a) You can solve the equation $2x = \frac{3}{4}$ using a diagram or algebraically.

Strategies

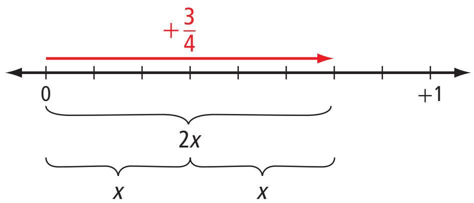
Draw a Diagram

Method 1: Use a Diagram

Model the equation $2x = \frac{3}{4}$ on a number line.



The length of the curly bracket represents $2x$, so half of this length represents x.



Why were the divisions on the number line changed from quarters to eighths?

The diagram shows that $x = \frac{3}{8}$.

Method 2: Solve Algebraically

Solve by applying the opposite operation.

$$
\begin{aligned}
2x &= \frac{3}{4} \\
2x \div 2 &= \frac{3}{4} \div 2 \\
x &= \frac{3}{4} \times \frac{1}{2} \\
&= \frac{3}{8}
\end{aligned}
$$

Why do you divide both sides by 2?

To divide $\frac{3}{4}$ by 2, why do you multiply by $\frac{1}{2}$?

Literacy Link

An *opposite operation* "undoes" another operation. Examples of opposite operations are:

- addition and subtraction
- multiplication and division

Opposite operations are also called *inverse operations*.

Check:

Left Side $= 2x$ Right Side $= \frac{3}{4}$

$$
\begin{aligned}
&= 2\left(\frac{3}{8}\right) \\
&= \frac{6}{8} \\
&= \frac{3}{4}
\end{aligned}
$$

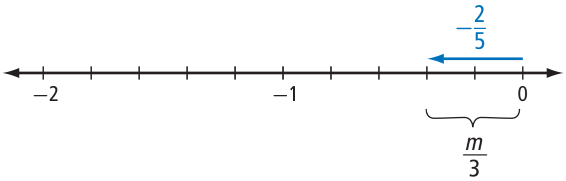
Left Side = Right Side

The solution, $x = \frac{3}{8}$, is correct.

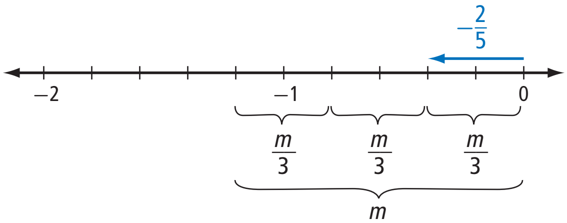
b) You can solve the equation $\frac{m}{3} = -\frac{2}{5}$ using a diagram or algebraically.

Method 1: Use a Diagram

Model the equation $\frac{m}{3} = -\frac{2}{5}$ on a number line.



The length of the curly bracket represents $\frac{m}{3}$, so use three of these to represent m.



The diagram shows that $m = -\frac{6}{5}$, or $-1\frac{1}{5}$.

Method 2: Solve Algebraically

Solve by applying the opposite operation.

$$\frac{m}{3} = -\frac{2}{5}$$

$$3 \times \frac{m}{3} = 3 \times \left(-\frac{2}{5}\right)$$

$$m = \frac{3}{1} \times \frac{-2}{5}$$

$$= \frac{-6}{5} \text{ or } -\frac{6}{5} \text{ or } -1\frac{1}{5}$$

What is the opposite operation of dividing by 3?

$$3 \times \left(-\frac{2}{5}\right) \approx 3 \times \left(-\frac{1}{2}\right)$$
$$\approx -1\frac{1}{2}$$

ME

$-1\frac{1}{5}$ is close to the estimate, so this answer is reasonable.

Check:

Left Side $= \frac{m}{3}$ Right Side $= -\frac{2}{5}$

$$= \frac{-\frac{6}{5}}{3}$$

$$= -\frac{6}{5} \div 3$$

$$= \frac{-6}{5} \times \frac{1}{3}$$

$$= \frac{-6}{15}$$

$$= \frac{-2}{5} \text{ or } -\frac{2}{5}$$

Left Side = Right Side

The solution, $m = -\frac{6}{5}$, is correct.

c) Isolate the variable by applying the opposite operation.

$$
\begin{aligned}
-2\tfrac{1}{2}k &= -3\tfrac{1}{2} \\
-2\tfrac{1}{2}k \div \left(-2\tfrac{1}{2}\right) &= -3\tfrac{1}{2} \div \left(-2\tfrac{1}{2}\right) \\
k &= -\frac{7}{2} \div \left(-\frac{5}{2}\right) \\
&= \frac{-7}{2} \div \frac{-5}{2} \\
&= \frac{-7}{-5} \\
&= \frac{7}{5} \text{ or } 1\tfrac{2}{5}
\end{aligned}
$$

What is the sign of the quotient when a negative is divided by a negative?

$-4 \div (-3) = \frac{4}{3}$ ME

One way to divide fractions with the same denominator is to simply divide the numerators. How else could you divide these fractions?

Check:

$$
\begin{aligned}
\text{Left Side} &= -2\tfrac{1}{2}k \qquad\qquad \text{Right Side} = -3\tfrac{1}{2} \\
&= -2\tfrac{1}{2}\left(1\tfrac{2}{5}\right) \\
&= -\frac{5}{2}\left(\frac{7}{5}\right) \\
&= -\frac{35}{10} \\
&= -\frac{7}{2} \text{ or } -3\tfrac{1}{2}
\end{aligned}
$$

Left Side = Right Side

The solution, $k = \frac{7}{5}$, is correct.

Show You Know

Solve.

a) $3x = -\frac{2}{3}$

b) $\frac{x}{2} = \frac{5}{6}$

c) $-1\tfrac{1}{4}y = 1\tfrac{3}{4}$

Example 2: Solve One-Step Equations With Decimals

Solve each equation. Check each solution.

a) $-1.2x = -3.96$ **b)** $\frac{r}{0.28} = -4.5$

Solution

a) Solve by applying the opposite operation.

$$-1.2x = -3.96$$
$$\frac{-1.2x}{-1.2} = \frac{-3.96}{-1.2}$$
$$x = 3.3$$

C 3.96 +/− ÷ 1.2 +/− = 3.3

$-4 \div -1 = 4$ ME

Check:

Left Side $= -1.2x$ Right Side $= -3.96$

$= -1.2(3.3)$

$= -3.96$

Left Side = Right Side

C 1.2 +/− × 3.3 = -3.96

The solution, $x = 3.3$, is correct.

b) Isolate the variable by applying the opposite operation.

$$\frac{r}{0.28} = -4.5$$
$$0.28 \times \frac{r}{0.28} = 0.28 \times (-4.5)$$
$$r = -1.26$$

C 0.28 × 4.5 +/− = -1.26

$0.28 \times 4.5 \approx 0.3 \times 4$
≈ 1.2 ME

What is the sign of the product when a positive is multiplied by a negative?

Check:

Left Side $= \frac{r}{0.28}$ Right Side $= -4.5$

$= \frac{-1.26}{0.28}$

$= -4.5$

Left Side = Right Side

C 1.26 +/− ÷ 0.28 = -4.5

The solution, $r = -1.26$, is correct.

Show You Know

Solve and check.

a) $\frac{u}{1.3} = 0.8$

b) $5.5k = -3.41$

Example 3: Apply Equations of the Form $\frac{a}{x} = b$

The formula for speed is $s = \frac{d}{t}$, where s is speed, d is distance, and t is time. The length of a Canadian football field, including the end zones, is 137.2 m. If a horse gallops at 13.4 m/s, how much time would it take the horse to gallop the length of the field? Express your answer to the nearest tenth of a second.

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Did You Know?

At Calgary Stampeders football games, a white horse named Quick Six charges the length of the field each time the Stampeders score a touchdown. The rider, Karyn Drake, carries the team flag.

Solution

Substitute the known values into the formula.

$$
\begin{aligned}
s &= \frac{d}{t} \\
13.4 &= \frac{137.2}{t} \\
t \times 13.4 &= t \times \frac{137.2}{t} \\
t \times 13.4 &= 137.2 \\
\frac{t \times 13.4}{13.4} &= \frac{137.2}{13.4} \\
t &\approx 10.2
\end{aligned}
$$

Why do you multiply by t?

$130 \div 13 = 10$

C 137.2 ÷ 13.4 = 10.23880597

The horse would take approximately 10.2 s to gallop the length of the field.

Check:
For a word problem, check your answer by verifying that the solution is consistent with the information given in the problem.

Calculate the speed by dividing the distance, 137.2 m, by the time, 10.2 s.

C 137.2 ÷ 10.2 = 13.45098039

Because the time was not exactly 10.2 s, this calculated speed of about 13.45 m/s is not exactly the same as the speed of 13.4 m/s given in the problem. But since these speed values are close, the answer is reasonable.

Show You Know

If a musher and her dog-team average 23.5 km/h during a dogsled race, how long will it take to sled 50 km? Express your answer to the nearest tenth of an hour.

Example 4: Write and Solve Equations

Winter Warehouse has winter jackets on sale at 25% off the regular price. If a jacket is on sale for \$176.25, what is the regular price of the jacket?

Solution

Let p represent the regular price of the jacket.

The sale price is 75% of the regular price.
So, the sale price is $0.75p$.

How do you know that the sale price is 75% of the regular price?

Since the sale price is \$176.25, an equation that represents the situation is

$$0.75p = 176.25$$

$$\frac{0.75p}{0.75} = \frac{176.25}{0.75}$$

$$p = 235$$

200 ÷ 1 = 200 ME

C 176.25 ÷ 0.75 = 235.

The regular price of the jacket is \$235.

Check:
The price reduction is 25% of \$235.

$0.25 \times \$235 = \$58.75$

The sale price is \$235 − \$58.75.

$\$235 - \$58.75 = \$176.25$

The calculated sale price agrees with the value given in the problem, so the answer, \$235, is correct.

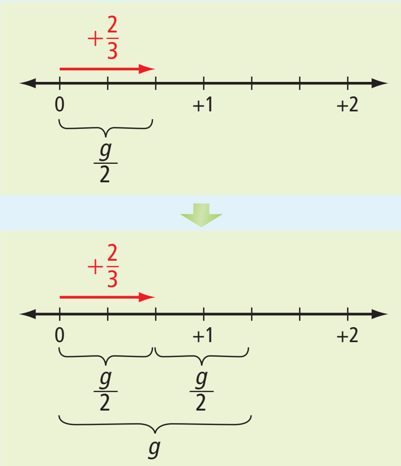
Show You Know

Winter Warehouse is selling mitts at 30% off the regular price.
If the sale price is \$34.99, what is the regular price of the mitts?

Key Ideas

- You can solve equations in various ways, including
  - using diagrams

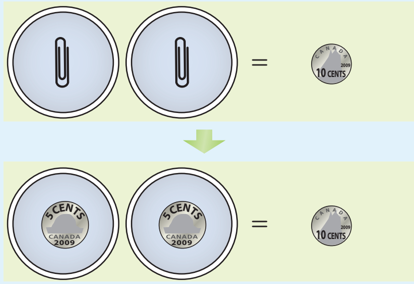
$$\frac{g}{2} = \frac{2}{3}$$



$$g = \frac{4}{3} \text{ or } 1\frac{1}{3}$$

  - using concrete materials

$2x = 0.10$



$x = 0.05$

  - using an algebraic method

$$\frac{-1.4}{p} = -0.8$$
$$p \times \left(\frac{-1.4}{p}\right) = p \times (-0.8)$$
$$-1.4 = p \times (-0.8)$$
$$\frac{-1.4}{-0.8} = \frac{p \times (-0.8)}{-0.8}$$
$$1.75 = p$$

- You can check solutions by using substitution.

Left Side $= \frac{-1.4}{p}$ Right Side $= -0.8$

$$= \frac{-1.4}{1.75}$$
$$= -0.8$$

C 1.4 +/− ÷ 1.75 = -0.8

Left Side = Right Side

The solution, $p = 1.75$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. To solve the equation $\frac{y}{2} = \frac{5}{3}$, John first multiplied both sides by 3.

a) Do you think that John's first step is the best way to isolate the variable y? Explain.

b) How would you solve the equation?

2. When Ming solved $0.3g = 0.8$, her value for g was 2.666… . She expressed this to the nearest tenth, or 2.7. When Ming checked by substitution, she found that the left side and the right side did not exactly agree.

$$\begin{aligned}\text{Left Side} &= 0.3g \\ &= 0.3(2.7) \\ &= 0.81\end{aligned} \qquad \text{Right Side} = 0.8$$

a) How could Ming make the left side and right side agree more closely?

b) Did Ming's check show that the solution was correct? Explain.

3. The length of Shamika's stride is 0.75 m. Both Amalia and Gustav were trying to calculate how many strides it would take Shamika to walk 30 m from her home to the bus stop.

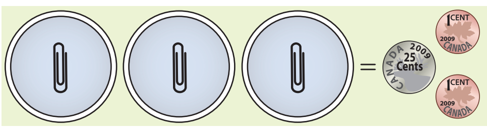
a) Amalia represented the situation with the equation $0.75p = 30$. Explain her thinking.

b) Gustav represented the situation with the equation $\frac{30}{p} = 0.75$. Explain his thinking.

c) Whose equation would you prefer to use? Explain.

Practise

4. Write an equation that is represented by the model shown. Then, solve it.



For help with #5 to #7, refer to Example 1 on pages 294–296.

5. Model the solution to the equation $4x = \frac{3}{4}$ using a number line.

6. Solve.

a) $2v = \frac{-5}{6}$ **b)** $\frac{x}{2} = \frac{2}{5}$

c) $\frac{4}{3} = -1\frac{1}{4}a$ **d)** $-1\frac{1}{2}x = -2\frac{1}{4}$

7. Solve.

a) $\frac{3}{5} = \frac{x}{4}$ **b)** $2y = -\frac{6}{5}$

c) $-\frac{7}{6} = -\frac{4}{3}n$ **d)** $2\frac{2}{3}w = 1\frac{1}{6}$

For help with #8 and #9, refer to Example 2 on page 297.

8. Solve and check.

a) $-5.6x = 3.5$ **b)** $\frac{e}{-2.2} = -0.75$

9. Solve.

a) $\frac{h}{4.1} = 3.6$ **b)** $1.472 = 0.46c$

For help with #10 and #11, refer to Example 3 on page 298.

10. Solve and check.

a) $-5.5 = \dfrac{1.1}{a}$ **b)** $\dfrac{4.8}{m} = 6.4$

11. Solve. Express each solution to the nearest hundredth.

a) $\dfrac{2.02}{n} = 0.71$ **b)** $-7.8 = \dfrac{-4.3}{x}$

Apply

12. The average speed of a vehicle, s, is represented by the formula $s = \dfrac{d}{t}$ where d is the distance driven and t is the time.

a) If Pablo drove at an average speed of 85 km/h for 3.75 h, what distance did he drive?

b) If Sheila drove 152 km at an average speed of 95 km/h, how much time did her trip take?

13. A roll of nickels is worth \$2.00. Write and solve an equation to determine the number of nickels in a roll.

14. Write and solve an equation to determine the side length, s, of a square with a perimeter of 25.8 cm.

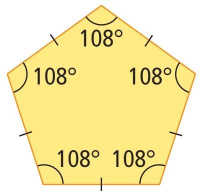
15. Without solving the equation $-\dfrac{5}{d} = -1.3$, predict whether d is greater than or less than 0. Explain your reasoning.

16. The diameter, d, of a circle is related to the circumference, C, by the formula $\dfrac{C}{d} = \pi$. Calculate the diameter of a circle with a circumference of 54.5 cm. Express your answer to the nearest tenth of a centimetre.

17. A regular polygon has a perimeter of 34.08 cm and a side length of 5.68 cm. Identify the polygon.

Literacy Link

A *regular polygon* has equal sides and equal angles. For example, a regular pentagon has five equal sides. Each angle measures 108°.



18. One year, a student council sold 856 copies of a school yearbook. Four fifths of the students at the school bought a copy. How many students did not buy a yearbook?

19. A score of 17 on a math test results in a mark of 68%. What score would give you a mark of 100%?

20. The area of Nunavut is about $4\frac{3}{8}$ times the area of the Yukon Territory. Nunavut covers 21% of Canada's area. What percent of Canada's area does the Yukon Territory cover?

Lewis Lake, Yukon

21. Dianne spends 40% of her net income on rent and 15% of her net income on food. If she spends a total of \$1375 per month on rent and food, what is her net monthly income?

22. Ellen and Li play on the same basketball team. In one game, Ellen scored one tenth of the team's points and Li scored one fifth of the team's points. Together, Ellen and Li scored 33 points. How many points did the team score altogether?

23. Sailaway Travel has a last-minute sale on a Caribbean cruise at 20% off. Their advertisement reads, "You save \$249.99." What is the sale price of the cruise?

24. Organizers of the Canadian Francophone Games hope to attract 500 volunteers to help host the games. The organizers predict that there will be about three and half times as many experienced volunteers as first-time volunteers. About how many first-time volunteers do they expect to attract?

25. A square piece of paper is folded in half to make a rectangle. The perimeter of the rectangle is 24.9 cm. What is the side length of the square piece of paper?

Extend

26. Solve.

a) $\frac{1}{3} + \frac{1}{6} = \frac{5}{6}x$

b) $-\frac{0.45}{1.8} = -\frac{0.81}{z}$

c) $\frac{y}{4} - \frac{y}{3} = -\frac{1}{10}$

d) $\frac{f}{0.55} = 2.6 - 3.5$

27. Solve. Express each solution to the nearest hundredth.

a) $0.75 + 1.23 = -3.9t$

b) $\frac{6.3}{h} = 2(-4.05)$

28. Solve and check.

a) $x \div \frac{1}{2} = -\frac{3}{4}$

b) $t \div \left(-\frac{2}{3}\right) = -\frac{1}{2}$

c) $\frac{5}{6} \div y = \frac{2}{3}$

d) $-\frac{2}{5} \div g = \frac{3}{10}$

29. a) A jar contains equal numbers of nickels and dimes. The total value of the coins is \$4.05. How many coins are in the jar?

b) A jar contains a mixture of nickels and dimes worth a total of \$4.75. There are three times as many nickels as dimes. How many dimes are there?

30. A cyclist is travelling six times as fast as a pedestrian. The difference in their speeds is 17.5 km/h. What is the cyclist's speed?

Math Link

Solve parts a) and b) in at least two different ways. Write and solve an equation as one of the methods for each part. Share your solutions with your classmates.

Three dried figs contain about 1.2 mg of iron.

a) What is the mass of iron in one dried fig?

b) Teenagers need about 12 mg of iron per day. How many dried figs would you have to eat to get your recommended daily amount of iron?

c) Write a formula that relates the mass of iron to the number of figs. Use your equation to calculate the mass of iron in eight figs.

d) Use your formula in part c) to determine the number of figs that contain 1.8 mg of iron.

8.2 Solving Equations: $ax + b = c$, $\frac{x}{a} + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations involving two operations
- solve linear equations with rational numbers using two operations

Web Link

To learn more about waterfalls in Canada and around the world, go to www.mathlinks9.ca and follow the links.

Two of Canada's highest measured waterfalls are in British Columbia. Takakkaw Falls, is in Yoho National Park, 27 km west of Lake Louise. Its height is 254 m. This is 34 m more than half the height of Della Falls in Strathcona Park on Vancouver Island.

Choose a variable to represent the height of Della Falls. Then, write and solve an equation to find the height of Della Falls.

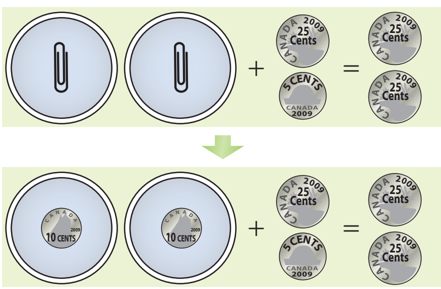
Della Falls, Vancouver Island

Materials

- coins
- cups or small containers
- paper clips

Explore Equations With Two Operations

1. a) How does the diagram model the solution to the equation $2x + 0.30 = 0.50$?
   b) What is the solution?



2. a) Explain how the second part of the diagram in #1 can model the equation $0.10y + 0.30 = 0.50$. What is the solution? Explain.

b) How does the second part of the diagram in #1 also model the solution to the equation $\frac{z}{10} + \frac{3}{10} = \frac{1}{2}$? What is the solution?

3. Describe how you would use manipulatives or diagrams to model the solution to each of the following.

a) $3x + 0.05 = 0.26$

b) $0.01x + 0.05 = 0.08$

c) $\frac{x}{4} + \frac{1}{5} = \frac{7}{10}$

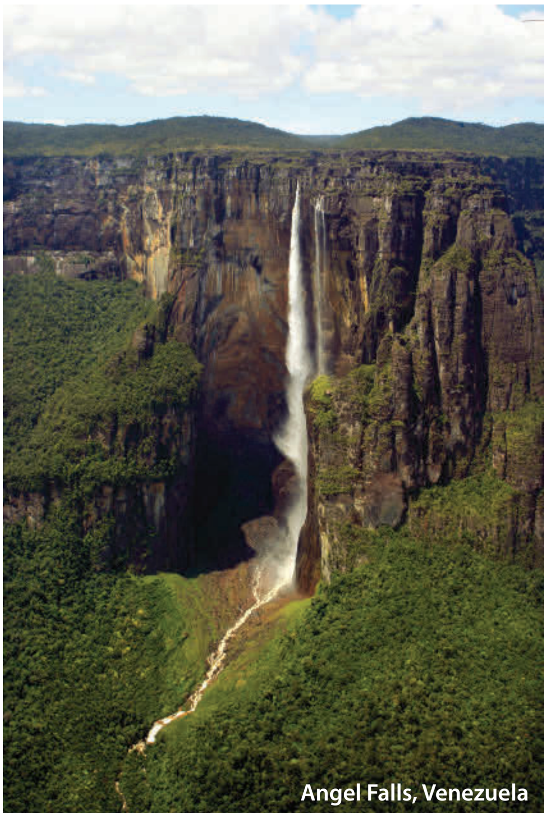
4. Work with a partner to explore how to model the solution to the equation $2x - 0.11 = 0.15$. Share your models with other classmates.

Reflect and Check

5. a) How can you model solutions to equations of the form $ax + b = c$ and $\frac{x}{a} + b = c$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

6. The tallest waterfall in the world is Angel Falls in Venezuela, with a height of about 0.8 km. This height is 0.08 km less than twice the height of Della Falls. Write and solve an equation to determine the height of Della Falls in kilometres. Check that your answer agrees with the height in metres you determined at the beginning of this section.



Angel Falls, Venezuela

Did You Know?

Canada's tallest free-standing structure is the CN Tower, with a height of about 550 m. This is about 250 m less than the height of Angel Falls.

Link the Ideas

Example 1: Solve Two-Step Equations With Fractions

Solve and check.

a) $2x + \frac{1}{10} = \frac{3}{5}$ **b)** $\frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$

Solution

a)

$$2x + \frac{1}{10} = \frac{3}{5}$$

$$2x + \frac{1}{10} - \frac{1}{10} = \frac{3}{5} - \frac{1}{10}$$

$$2x = \frac{3}{5} - \frac{1}{10}$$

$$2x = \frac{6}{10} - \frac{1}{10}$$

$$2x = \frac{5}{10}$$

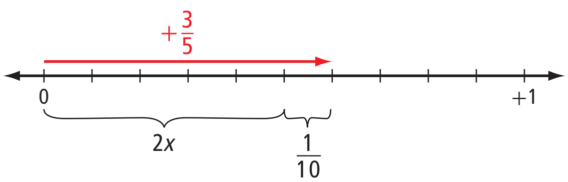
$$2x \div 2 = \frac{1}{2} \div 2$$

$$x = \frac{1}{4}$$

To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

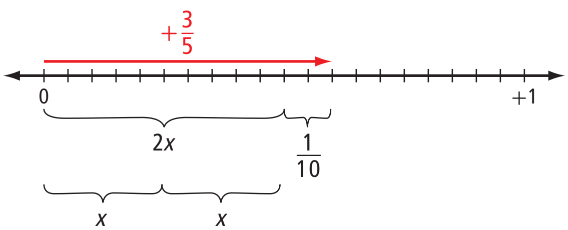
$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2}$

Check by modelling the equation $2x + \frac{1}{10} = \frac{3}{5}$ on a number line.



This diagram models the original equation $2x + \frac{1}{10} = \frac{3}{5}$. How does it show that $2x = \frac{5}{10}$?

Now show the value of x.



The second diagram shows that $x = \frac{5}{20}$ or $\frac{1}{4}$.

The solution, $x = \frac{1}{4}$, is correct.

b) $\frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$

$\frac{k}{3} - \frac{1}{2} = -\frac{7}{4}$

You may prefer to work with integers than to perform fraction operations. Change from fractions to integers by multiplying by a common multiple of the denominators.

$$12 \times \frac{k}{3} - 12 \times \frac{1}{2} = 12 \times \left(-\frac{7}{4}\right)$$
$$4k - 6 = -21$$
$$4k - 6 + 6 = -21 + 6$$
$$4k = -15$$
$$\frac{4k}{4} = \frac{-15}{4}$$
$$k = -\frac{15}{4}$$

A common multiple of the denominators 3, 2, and 4 is 12.

Check:

Left Side $= \frac{k}{3} - \frac{1}{2}$ Right Side $= -1\frac{3}{4}$

$$= -\frac{15}{4} \div 3 - \frac{1}{2}$$
$$= \frac{-15}{4} \times \frac{1}{3} - \frac{1}{2}$$
$$= \frac{-5}{4} - \frac{1}{2}$$
$$= \frac{-5}{4} - \frac{2}{4}$$
$$= \frac{-7}{4} \text{ or } -1\frac{3}{4}$$

Left Side = Right Side

The solution, $k = -\frac{15}{4}$, is correct.

Show You Know

Solve and check.

a) $2y + \frac{1}{2} = \frac{3}{4}$

b) $\frac{n}{2} - \frac{3}{4} = 2\frac{3}{8}$

Example 2: Solve Two-Step Equations With Decimals

Solve $\frac{a}{2.8} - 2.5 = -3.7$ and check the solution.

Solution

$$\frac{a}{2.8} - 2.5 = -3.7$$
$$\frac{a}{2.8} - 2.5 + 2.5 = -3.7 + 2.5$$
$$\frac{a}{2.8} = -1.2$$
$$2.8 \times \frac{a}{2.8} = 2.8 \times (-1.2)$$
$$a = -3.36$$

$2.8 \times (-1.2) \approx 3 \times (-1)$
≈ -3
ME

Check:

Left Side $= \frac{a}{2.8} - 2.5$ Right Side $= -3.7$

$$= \frac{-3.36}{2.8} - 2.5$$
$$= -1.2 - 2.5$$
$$= -3.7$$

Left Side = Right Side

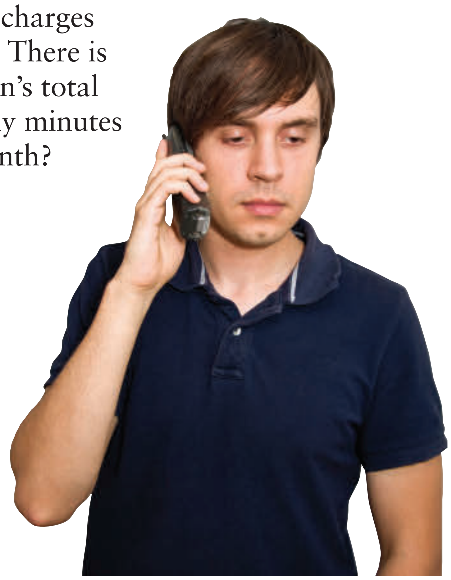
The solution, $a = -3.36$, is correct.

Show You Know

Solve $\frac{h}{1.6} + 3.3 = 1.8$ and check the solution.

Example 3: Apply Two-Step Equations With Decimals

Colin has a long-distance telephone plan that charges 5¢/min for long-distance calls within Canada. There is also a monthly fee of \$4.95. One month, Colin's total long-distance charges were \$18.75. How many minutes of long-distance calls did Colin make that month?



Solution

Let m represent the unknown number of minutes.

The cost per minute is 5¢ or \$0.05.
The cost of the phone calls, in dollars, is $0.05m$.
The total cost for the month is the cost of the calls plus the monthly fee, or $0.05m + 4.95$.
The total cost for the month is \$18.75.

An equation that represents the situation is $0.05m + 4.95 = 18.75$.

$$
\begin{aligned}
0.05m + 4.95 \color{green}{- 4.95} &= 18.75 \color{green}{- 4.95} \\
0.05m &= 13.80 \\
\frac{0.05m}{\color{green}{0.05}} &= \frac{13.80}{\color{green}{0.05}} \\
m &= 276
\end{aligned}
$$

$$
\begin{aligned}
\frac{13.80}{0.05} &\approx \frac{15}{0.05} \\
&\approx 300
\end{aligned}
$$

Colin made 276 min of long-distance calls that month.

Check:
The cost for 276 min at 5¢/min is $\$0.05 \times 276$.
$0.05 \times 276 = 13.80$
The total cost for the month is $\$13.80 + \$4.95$, which equals $\$18.75$.
This total cost agrees with the value stated in the problem.

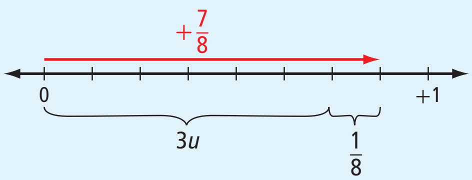
Show You Know

Suppose that Colin changes to a cheaper long-distance plan. This plan charges 4¢/min for long-distance calls within Canada, plus a monthly fee of $3.95. For how many minutes could he call long distance in a month for the same total long-distance charge of $18.75?

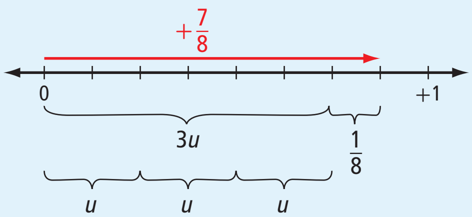
Key Ideas

- You can determine or check some solutions by using a model.

$3u + \frac{1}{8} = \frac{7}{8}$



$3u = \frac{6}{8}$



$u = \frac{2}{8}$ or $\frac{1}{4}$

- To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

$$
\begin{aligned}
0.4w - 1.5 &= 0.3 \\
0.4w - 1.5 \color{green}{+ 1.5} &= 0.3 \color{green}{+ 1.5} \\
0.4w &= 1.8 \\
\frac{0.4w}{\color{green}{0.4}} &= \frac{1.8}{\color{green}{0.4}} \\
w &= 4.5
\end{aligned}
$$

- To solve two-step equations involving fractions, you may prefer to rewrite the equation and work with integers than to perform fraction operations.

$$\frac{w}{5} - \frac{3}{2} = \frac{1}{10}$$

To work with integers, multiply all terms by a common multiple of the denominators. For the denominators 5, 2, and 10, a common multiple is 10.

$$\begin{aligned}
10 \times \frac{w}{5} - 10 \times \frac{3}{2} &= 10 \times \frac{1}{10} \\
2w - 15 &= 1 \\
2w - 15 + 15 &= 1 + 15 \\
2w &= 16 \\
\frac{2w}{2} &= \frac{16}{2} \\
w &= 8
\end{aligned}$$

- You can check solutions by using substitution.

$$\begin{aligned}
\text{Left Side} &= \frac{w}{5} - \frac{3}{2} \qquad\qquad \text{Right Side} = \frac{1}{10} \\
&= \frac{8}{5} - \frac{3}{2} \\
&= \frac{16}{10} - \frac{15}{10} \\
&= \frac{1}{10}
\end{aligned}$$

Left Side = Right Side

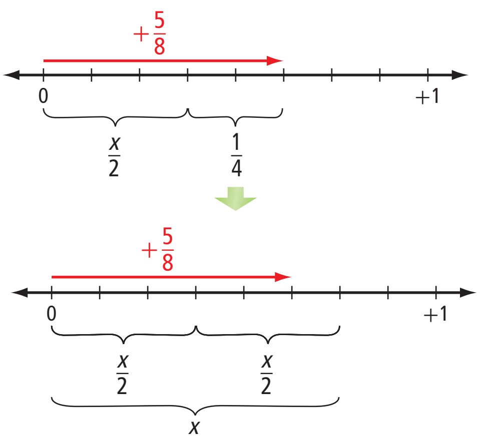
The solution, $w = 8$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Explain how the diagrams model the equation $\frac{x}{2} + \frac{1}{4} = \frac{5}{8}$ and its solution. What is the solution?



2. Ryan solved $2r + 0.3 = 0.7$ as follows. Do you agree with his solution? Explain.

$$\frac{2r}{2} + 0.3 = \frac{0.7}{2}$$
$$r + 0.3 = 0.35$$
$$r + 0.3 - 0.3 = 0.35 - 0.3$$
$$r = 0.05$$

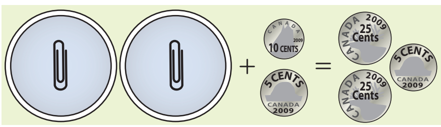
3. Jenna did not want to perform fraction operations to solve the equation $\frac{x}{2} - \frac{1}{9} = \frac{5}{6}$, so she first multiplied both sides by 54. Is this the common multiple you would have chosen? Explain.

4. a) Milos solved $0.05x - 0.12 = 0.08$ by multiplying both sides by 100 and then solving $5x - 12 = 8$. Show how he used this method to determine the correct solution.

b) When Milos was asked to solve $\frac{x}{0.05} - 0.12 = 0.08$, he reasoned that he could determine the correct solution by solving $\frac{x}{5} - 12 = 8$. Do you agree with his reasoning? Explain.

Practise

5. Write an equation that is modelled by the following. Then, solve it.



6. Model the equation $3x + 0.14 = 0.50$ using concrete materials. Solve using your model.

For help with #7 and #8, refer to Example 1 on pages 306–307.

7. Solve.

a) $4y - \frac{2}{5} = \frac{3}{5}$

b) $2d - \frac{1}{2} = \frac{5}{4}$

c) $\frac{n}{2} + 1\frac{2}{3} = \frac{1}{6}$

d) $\frac{4}{5} - 2\frac{1}{2}r = \frac{3}{10}$

8. Solve.

a) $1\frac{1}{2} = 4h + \frac{2}{3}$

b) $\frac{4}{3}x + \frac{3}{4} = \frac{1}{2}$

c) $\frac{3}{4} - \frac{d}{3} = \frac{3}{8}$

d) $-4\frac{2}{5} = -3\frac{1}{5} + \frac{7}{10}g$

For help with #9 and #10, refer to Example 2 on page 308.

9. Solve and check.

a) $\frac{x}{0.6} + 2.5 = -1$

b) $0.38 = 6.2 - \frac{r}{1.2}$

10. Solve.

a) $-0.02 - \frac{n}{3.7} = -0.01$

b) $\frac{k}{-0.54} + 0.67 = 3.47$

For help with #11 and #12, refer to Example 3 on pages 308–309.

11. Solve and check.

a) $2 + 12.5v = 0.55$

b) $-0.77 = -0.1x - 0.45$

12. Solve.

a) $0.074d - 3.4 = 0.707$

b) $67 = 5.51 + 4.3a$

Apply

13. The cost of a pizza is \$8.50 plus \$1.35 per topping. How many toppings are on a pizza that costs \$13.90?

14. Hiroshi paid \$34.95 to rent a car for a day, plus 12¢ for each kilometre he drove. The total rental cost, before taxes, was \$55.11. How far did Hiroshi drive that day?

15. On Saturday morning, Marc had a quarter of his weekly allowance left. He spent a total of \$6.50 on bus fares and a freshly squeezed orange juice on Saturday afternoon. He then had \$2.25 left. How much is his weekly allowance?

16. Nadia has a summer job in an electronics store. She is paid \$400 per week, plus 5% commission on the total value of her sales.

a) One week, when the store was not busy, Nadia earned only \$510.30. What was the total value of her sales that week?

b) Nadia's average earnings are \$780 per week. What is the average value of her weekly sales?

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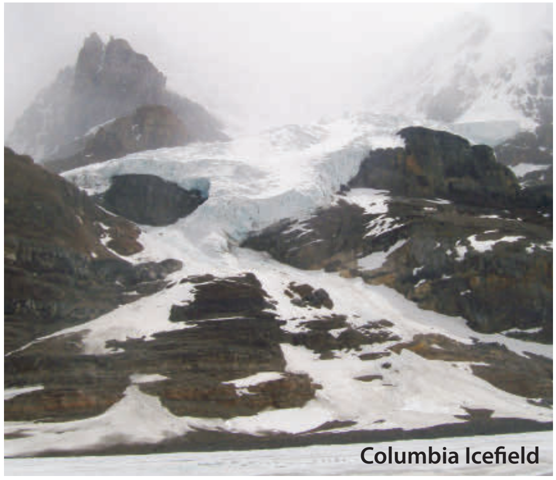
17. Benoit was helping his family build a new fence along one side of their yard. The total length of the fence is 28 m. They worked for two days and completed an equal length of fence on each day. On the third day, they completed the remaining 4.8 m of fence. What length of fence did they build on each of the first two days?

18. The perimeter of a regular hexagon is 3.04 cm less than the perimeter of a regular pentagon. The perimeter of the regular hexagon is 21.06 cm. What is the side length of the regular pentagon?

19. The greatest average annual snowfall in Alberta is on the Columbia Icefield. The greatest average annual snowfall in Manitoba is at Island Lake. An average of 642.9 mm of snow falls on the Columbia Icefield in a year. This amount of snow is 22.5 mm less than twice the annual average at Island Lake. What is the average annual snowfall at Island Lake?


Columbia Icefield

20. During a camping trip, Nina was making a lean-to for sleeping. She cut a 2.5-m long post into two pieces, so that one piece was 26 cm longer than the other. What was the length of each piece?

21. The average monthly rainfall in Victoria in July is 2.6 mm less than one fifth of the amount of rain that falls in Edmonton in the same period. Victoria averages 17.6 mm of rainfall in July. What is the average monthly rainfall in Edmonton in July?

22. The temperature in Winnipeg was 7 °C and was falling by 2.5 °C/h. How many hours did it take for the temperature to reach −5.5 °C?

23. Max and Sharifa are both saving to buy the same model of DVD player, which costs \$99, including tax. Max already has \$31.00 and decides to save \$8.50 per week from now on. Sharifa already has \$25.50 and decides to save \$10.50 per week from now on. Who can pay for the DVD player first? Explain.

24. A cylindrical storage tank that holds 375 L of water is completely full. A pump removes water at a rate of 0.6 L/s. For how many minutes must the pump work until 240 L of water remain in the tank?

25. The average distance of Mercury from the sun is 57.9 million kilometres. This distance is 3.8 million kilometres more than half the average distance of Venus from the sun. What is the average distance of Venus from the sun?

26. Create an equation of the form $\frac{x}{a} + b = c$ with each given solution. Compare your equations with your classmates' equations.

a) $\frac{2}{3}$

b) -0.8

27. Write a word problem that can be solved using an equation of the form $ax + b = c$. Include at least one decimal or fraction. Have a classmate solve your problem.

Extend

28. Solve.

a) $\frac{3}{2} + \frac{w}{4} = \frac{5}{6} - \frac{1}{2}$

b) $\frac{3}{4}\left(-\frac{2}{9}\right) = 4\frac{1}{2}x + \frac{1}{3}$

29. Solve. Express each solution to the nearest hundredth.

a) $0.75 + 0.16y + 0.2y = 0.34$

b) $\frac{-1.85}{0.74} = 2.22 - 0.57s$

30. Solve.

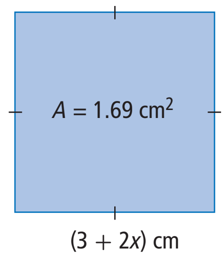
a) $\frac{0.2}{x} + 0.8 = 1.2$

b) $\frac{1}{2} - \frac{4}{n} = -\frac{1}{4}$

c) $-\frac{3.52}{h} - 1.31 = 1.19$

d) $4\frac{5}{6} = 3\frac{1}{3} - \frac{3}{y}$

31. Determine the value of x.



32. A freight train passes through a 750-m long tunnel at 50 km/h. The back of the train exits the tunnel 1.5 min after the front of the train enters it. What is the length of the train, in metres?


Revelstoke, British Columbia

Math Link

A slice of canned corned beef contains about 0.21 g of sodium. This much sodium is 0.01 g more than the mass of sodium in four slices of roast beef. What is the mass of sodium in a slice of roast beef?

a) Write an equation that models the situation.

b) Solve the equation in two different ways.

c) Which of your solution methods do you prefer? Explain.

8.3 Solving Equations: $a(x + b) = c$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations that include grouping symbols on one side
- solve linear equations that include grouping symbols on one side

Did You Know?

Farms account for only about 7% of the land in Canada. About 80% of Canada's farmland is located in the Prairie Provinces: Alberta, Saskatchewan, and Manitoba.

Each year, Canada's Prairie Provinces produce tens of millions of tonnes of grains, such as wheat, barley, and canola. The growth of a grain crop partly depends on the quantity of heat it receives. One indicator of the quantity of heat that a crop receives in a day is the *daily average temperature*. This is defined as the average of the high and low temperatures in a day.

How can you calculate the daily average temperature on a day when the high temperature is 23 °C and the low temperature is 13 °C? If the low temperature is 10 °C, how could you determine the high temperature that would result in a daily average temperature of 15 °C?

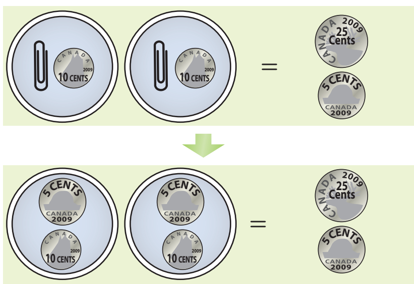
What equations can you use to represent these situations?

Materials

- coins
- paper cups or small containers
- paper clips

Explore Equations With Grouping Symbols

1. Explain how the diagram models the solution to the equation $2(x + 0.10) = 0.30$. What is the solution?



2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $2(x + 0.25) = 0.54$

b) $3\left(x + \frac{1}{20}\right) = \frac{9}{10}$

Reflect and Check

3. a) How can you model solutions to equations of the form $a(x + b) = c$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

4. Stefan works on a farm in the Fraser Valley of British Columbia. Two of the fields are square. The perimeter of the larger field is 2.4 km. The side length of the larger field is 0.1 km more than the side length of the smaller field. Create a labelled drawing of this situation. Suggest ways of determining the side length of the smaller field. Include a suggestion for solving an equation of the form $a(x + b) = c$. Share your ideas with your classmates.

Link the Ideas

Example 1: Solve Equations With Grouping Symbols

Solve and check.

a) $3(d + 0.4) = -3.9$

b) $\frac{t - 1}{5} = \frac{3}{2}$

Solution

a) *Method 1: Use the Distributive Property First*

Use the distributive property to remove the brackets.

$$\begin{aligned}
3(d + 0.4) &= -3.9 \\
(3 \times d) + (3 \times 0.4) &= -3.9 \\
3d + 1.2 &= -3.9 \\
3d + 1.2 - 1.2 &= -3.9 - 1.2 \\
3d &= -5.1 \\
\frac{3d}{3} &= \frac{-5.1}{3} \\
d &= -1.7
\end{aligned}$$

Literacy Link

The *distributive property* is:

$a(b + c) = ab + ac$

Method 2: Divide First

$$3(d + 0.4) = -3.9$$
$$\frac{3(d + 0.4)}{3} = \frac{-3.9}{3}$$
$$d + 0.4 = -1.3$$
$$d + 0.4 - 0.4 = -1.3 - 0.4$$
$$d = -1.7$$

Why do you divide both sides by 3?

Check:

Left Side $= 3(d + 0.4)$ Right Side $= -3.9$
$= 3(-1.7 + 0.4)$
$= 3(-1.3)$
$= -3.9$

Left Side = Right Side

The solution, $d = -1.7$, is correct.

b)
$$\frac{t-1}{5} = \frac{3}{2}$$
$$10 \times \frac{t-1}{5} = 10 \times \frac{3}{2}$$
$$2(t - 1) = 15$$
$$2t - 2 = 15$$
$$2t - 2 + 2 = 15 + 2$$
$$2t = 17$$
$$\frac{2t}{2} = \frac{17}{2}$$
$$t = \frac{17}{2}$$

Why do you multiply both sides by 10? Is there a different way to solve the equation?

Check:

Left Side $= \frac{t-1}{5}$ Right Side $= \frac{3}{2}$
$= \left(\frac{17}{2} - 1\right) \div 5$
$= \left(\frac{17}{2} - \frac{2}{2}\right) \div 5$
$= \frac{15}{2} \div \frac{5}{1}$
$= \frac{15}{2} \times \frac{1}{5}$
$= \frac{3}{2}$

Left Side = Right Side

The solution, $t = \frac{17}{2}$, is correct.

Literacy Link

A *fraction bar* acts as a grouping symbol and as a division symbol. The expression $\frac{t-1}{5}$ can be written as $\frac{1}{5}(t - 1)$ or as $(t - 1) \div 5$.

Show You Know

Solve and check.

a) $2(e - 0.6) = 4.2$

b) $\frac{c + 2}{3} = -\frac{5}{2}$

Example 2: Apply Equations With Grouping Symbols

On a typical February day in Whitehorse, Yukon Territory, the daily average temperature is −13.2 °C. The low temperature is −18.1 °C. What is the high temperature?

Solution

Let the high temperature be T degrees Celsius. The daily average temperature, in degrees Celsius, is the average of the high and low temperatures, or $\frac{T + (-18.1)}{2}$. The daily average temperature is −13.2 °C.

How could you estimate the high temperature?

An equation that represents this situation is $\frac{T + (-18.1)}{2} = -13.2$.

Isolate the variable, T.

$$\begin{aligned}
\frac{T + (-18.1)}{2} &= -13.2 \\
2 \times \frac{T + (-18.1)}{2} &= 2 \times (-13.2) \\
T - 18.1 &= -26.4 \\
T - 18.1 + 18.1 &= -26.4 + 18.1 \\
T &= -8.3
\end{aligned}$$

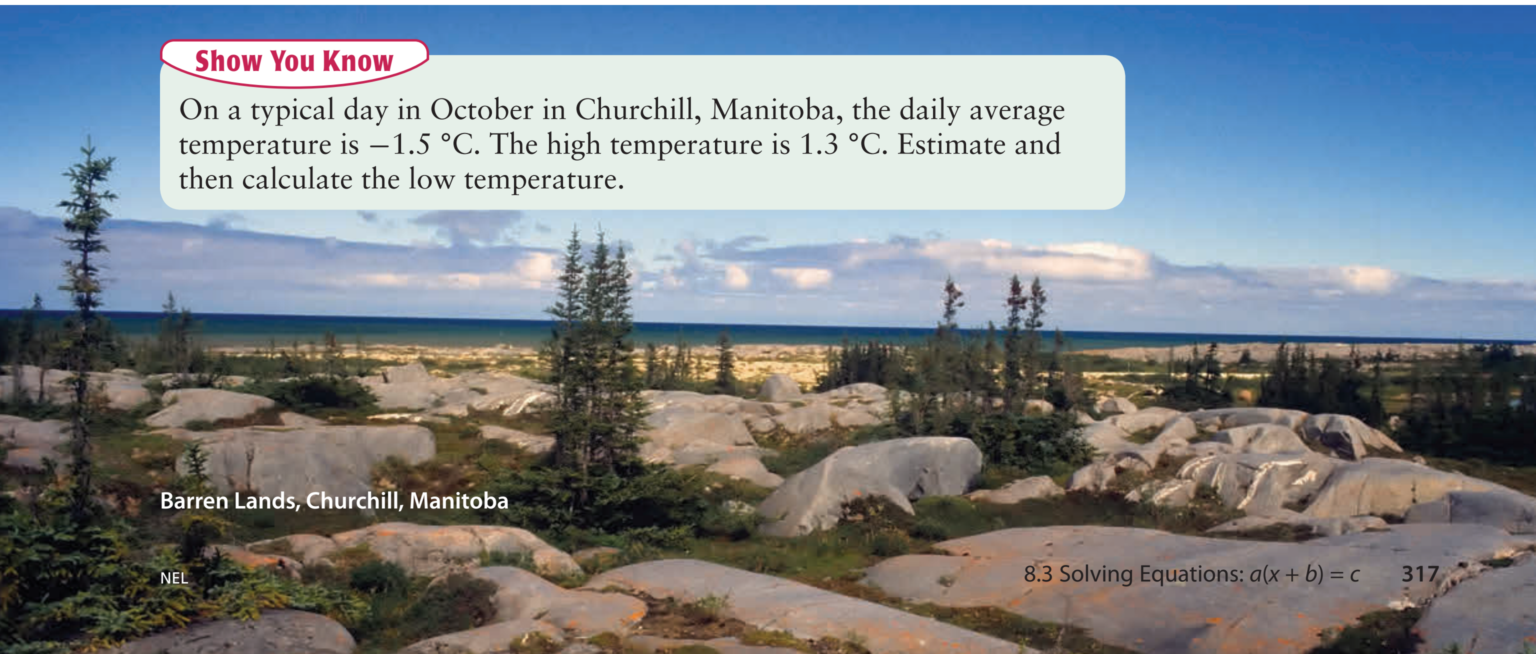
The high temperature is −8.3 °C.

Check:

The average of the high and low temperatures is $\frac{-8.3 + (-18.1)}{2}$.

$$\begin{aligned}
\frac{-8.3 + (-18.1)}{2} &= \frac{-26.4}{2} \\
&= -13.2
\end{aligned}$$

The calculated average of −13.2 °C agrees with the daily average temperature given in the problem.



Show You Know

On a typical day in October in Churchill, Manitoba, the daily average temperature is −1.5 °C. The high temperature is 1.3 °C. Estimate and then calculate the low temperature.

Barren Lands, Churchill, Manitoba

Key Ideas

- To isolate the variable in an equation of the form $a(x + b) = c$, you can
  - use the distributive property first

$$\begin{aligned} 4(r - 0.6) &= -3.2 \\ 4r - 2.4 &= -3.2 \\ 4r - 2.4 + 2.4 &= -3.2 + 2.4 \\ 4r &= -0.8 \\ \frac{4r}{4} &= \frac{-0.8}{4} \\ r &= -0.2 \end{aligned}$$

  - divide first

$$\begin{aligned} 4(r - 0.6) &= -3.2 \\ \frac{4(r - 0.6)}{4} &= \frac{-3.2}{4} \\ r - 0.6 &= -0.8 \\ r - 0.6 + 0.6 &= -0.8 + 0.6 \\ r &= -0.2 \end{aligned}$$

- To solve equations involving grouping symbols and fractions, you can rewrite the equation and work with integers instead of performing fraction operations.

$$\begin{aligned} \frac{q-1}{2} &= \frac{3}{4} \\ 4 \times \frac{q-1}{2} &= 4 \times \frac{3}{4} \\ 2(q - 1) &= 3 \\ 2q - 2 &= 3 \\ 2q - 2 + 2 &= 3 + 2 \\ 2q &= 5 \\ \frac{2q}{2} &= \frac{5}{2} \\ q &= \frac{5}{2} \end{aligned}$$

- You can check solutions by using substitution.

$$\begin{aligned} \text{Left Side} &= \frac{q-1}{2} \\ &= \left(\frac{5}{2} - 1\right) \div 2 \\ &= \left(\frac{5}{2} - \frac{2}{2}\right) \div 2 \\ &= \frac{3}{2} \times \frac{1}{2} \\ &= \frac{3}{4} \end{aligned}$$

$$\text{Right Side} = \frac{3}{4}$$

Left Side = Right Side

The solution, $q = \frac{5}{2}$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Mario solved the equation $2(n + 1.5) = 4.5$ as follows.

$$\begin{aligned} 2(n + 1.5) &= 4.5 \\ 2n + 3 &= 9 \\ 2n &= 6 \\ n &= 3 \end{aligned}$$

a) What is the error in his reasoning? Explain.

b) Write the correct solution.

2. Cal and Tyana solved the equation $3(k - 4.3) = -2.7$ in different ways. Cal used the distributive property first, while Tyana divided first.

a) Show both of their methods.

b) Whose method do you prefer? Explain.

c) If the equation was $3(k - 4.3) = -2.5$, which method would you use? Explain.

3. Renée and Paul used different methods to solve $\frac{x + 1}{2} = \frac{3}{5}$.

a) Renée first multiplied both sides by a common multiple.
$10 \times \frac{x + 1}{2} = 10 \times \frac{3}{5}$
Show the rest of her solution.

b) Paul first rewrote the equation as $\frac{1}{2}(x + 1) = \frac{3}{5}$ and used the distributive property. Show the rest of his solution.
$\frac{1}{2}x + \frac{1}{2} = \frac{3}{5}$

c) Which solution do you prefer? Explain.

4. Viktor and Ashni were solving the following problem together.

A square with a side length of $x + 1$ has a perimeter of 18.6 units. What is the value of x?

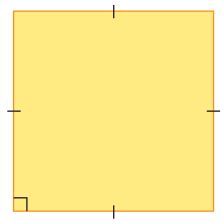
They disagreed over how to model the situation with an equation.

Viktor's Equation	*Ashni's Equation*
$4(x + 1) = 18.6$	$4x + 1 = 18.6$

a) Which equation is correct? Explain.

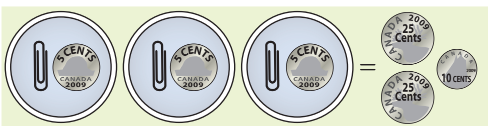
b) What is the value of x?

c) How can you check whether your value for x is correct?



Practise

5. Write an equation that is represented by the following. Then, solve the equation.



For help with #6 to #9, refer to Example 1 on pages 315–316.

6. Solve and check.

a) $2(x + 1.5) = 7.6$

b) $-2.8 = -1(c - 0.65)$

c) $-3.57 = 3(a + 4.51)$

d) $-3.6(0.25 - r) = 0.18$

7. Solve. Express each solution to the nearest hundredth.

a) $3(u - 12.5) = -3.41$

b) $14.01 = -7(1.93 + m)$

c) $6(0.15 + v) = 10.97$

d) $-9.5(x - 4.2) = 7.5$

8. Solve and check.

a) $\frac{n + 1}{2} = -\frac{3}{4}$

b) $\frac{5}{2} = \frac{1}{3}(x - 2)$

c) $\frac{3}{4}(w + 2) = 1\frac{1}{3}$

d) $\frac{7}{6} = \frac{2(5 - g)}{3}$

9. Solve.

a) $\frac{1 - y}{3} = \frac{2}{5}$

b) $-\frac{1}{2}(q + 4) = 2\frac{1}{4}$

c) $-\frac{7}{10} = \frac{e + 3}{5}$

d) $\frac{2(p - 3)}{3} = \frac{1}{2}$

For help with #10 and #11, refer to Example 2 on page 317.

10. Solve and check.

a) $\frac{x + 4.1}{3} = 2.5$

b) $19.8 = \frac{4.2 + k}{-3}$

c) $\frac{q - 6.95}{2} = -4.61$

d) $-2.1 = \frac{4.6 - a}{-5}$

11. Solve.

a) $-0.25 = \frac{q - 1.6}{2}$

b) $\frac{y + 0.385}{-1} = -0.456$

c) $\frac{7.34 + n}{4} = 1.29$

d) $7.56 = \frac{p - 15.12}{-2}$

Apply

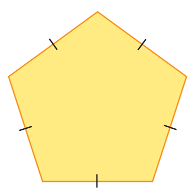
12. The mean of two numbers is 3.2. One of the numbers is 8.1. What is the other number?

13. Two equilateral triangles differ in their side lengths by 1.05 m. The perimeter of the larger triangle is 9.83 m. Determine the side length of the smaller triangle by

a) representing the situation with an equation of the form $a(x + b) = c$, and solving the equation.

b) using a different method of your choice. Explain your reasoning.

14. The regular pentagon has a perimeter of 18.8 units. What is the value of x?



15. On a typical January day in Prince Rupert, British Columbia, the daily average temperature is −0.2 °C. The low temperature is −3.7 °C. What is the high temperature?

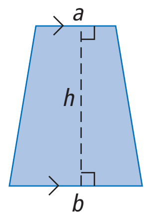
16. A regular hexagon has a perimeter of 41.4 units. The side length of the hexagon is represented by the expression $2(3 - d)$. What is the value of d?

17. Henri bought three jars of spaghetti sauce. He used a coupon that reduced the cost of each jar by \$0.75. If he paid \$6.72 altogether, what was the regular price of each jar?

18. Luisa bought five concert tickets. She paid a \$4.50 handling fee for each ticket. The total cost, before tax, was \$210.00. What was the cost of each ticket, excluding the handling fee?

19. Mary wants to make her family kamiks, which are boots made from seal or caribou skin. She usually pays \$80 for each skin, but Lukasie offers her a discount if she buys five skins. If Mary pays \$368, how much did Lukasie reduce the price of each skin?

20. The area of a trapezoid can be found using the formula $A = \frac{1}{2}(a + b) \times h$, where a and b are the lengths of the two parallel sides, and h is the distance between them.



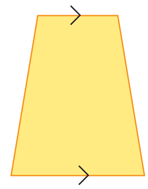
Determine each of the following.

a) h when $A = 27.3$ cm^2, $a = 2.3$ cm, and $b = 4.7$ cm

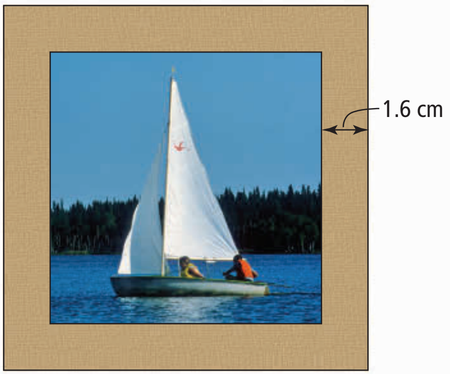
b) a when $A = 4.8$ m^2, $b = 1.9$ m, and $h = 3$ m

Literacy Link

A *trapezoid* is a quadrilateral with exactly two parallel sides.



21. A square picture frame is made from wood that is 1.6 cm wide. The perimeter of the outside of the frame is 75.2 cm. What is the side length of the largest square picture that the frame will display?



22. For a fit and healthy person, the maximum safe heart rate during exercise is approximately related to their age by the formula $r = \frac{4}{5}(220 - a)$. In this formula, r is the maximum safe heart rate in beats per minute, and a is the age in years. At what age is the maximum safe heart rate 164 beats/min?

Extend

23. Solve and check.

a) $2(x + 3) + 3(x + 2) = 0.5$

b) $4(y - 3) - 2(y + 1) = -4.2$

c) $1.5(4 + f) + 2.5(5 - f) = 15.7$

d) $-5.3 = 6.2(t + 6) - 1.2(t - 2)$

24. Solve.

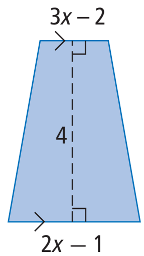
a) $4(d + 3) - 3(d - 2) = 1.2$

b) $-10.5 = 5(1 - r) + 4(r - 3)$

c) $3.9 = 2.5(g - 4) + 1.5(g + 5)$

d) $-1.8(h + 3) - 1.3(2 - h) = 1$

25. The area of the trapezoid is 1.5 square units. What are the lengths of the parallel sides?



26. If $1.5(x + 1) + 3.5(x + 1) = 7.5$, determine the value of $-10(x + 1)$ without determining the value of x. Explain your reasoning.

27. Tahir is training for an upcoming cross-country meet. He runs 13 km, three times a week. His goal is to increase his average speed by 1.5 km/h, so that he can complete each run in $1\frac{1}{4}$ h. How long does he take to complete each run now, to the nearest tenth of a minute? Solve this problem in two different ways.

28. a) Solve $x(n - 3) = 4$ for n by dividing first.

b) Solve $x(n - 3) = 4$ for n by using the distributive property first.

c) Which method do you prefer? Explain.

Math Link

One serving of a breakfast mixture consists of 200 mL of a corn bran cereal and 250 mL of 2% milk. Two servings of the mixture provide 1.4 mg of thiamin. If 250 mL of 2% milk provides 0.1 mg of thiamin, what mass of thiamin is in 200 mL of the cereal?

a) Write an equation that models the situation.

b) Solve the equation in two different ways.

c) Which of your solution methods do you prefer? Explain.

Did You Know?

Thiamin is another name for vitamin B_1. The body needs it to digest carbohydrates completely. A lack of thiamin can cause a loss of appetite, weakness, confusion, and even paralysis. Sources of thiamin include whole grains, liver, and yeast.

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations that include variables on both sides
- solve linear equations that include variables on both sides

Did You Know?

Greyhounds are the fastest dogs. They can run at about 72 km/h or 20 m/s.

Web Link

To learn more about the speeds of different animals, go to www.mathlinks9.ca and follow the links.

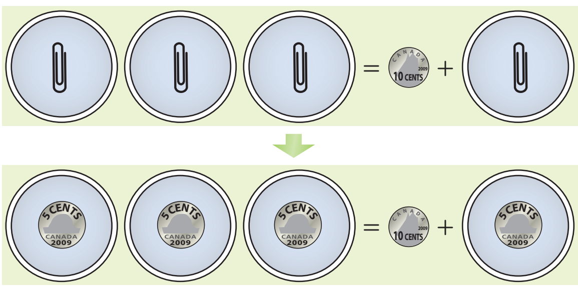
Laura is playing in the park with her greyhound, Dash. Laura picks up the ball and starts to run. When Dash starts chasing her, Laura is 20 m away from him and is running away at 5 m/s. Dash runs after her at 15 m/s. Suppose Dash chases Laura for t seconds. What situation does the equation $15t = 5t + 20$ represent? What does the expression on each side of the equal sign represent? If you solved this equation, what would the value of t indicate?

Materials

- coins
- paper cups or small containers
- paper clips

Explore Equations With Variables on Both Sides

1. Explain how the diagrams model the solution to the equation $3x = 0.10 + x$. What is the solution?



2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $3x + 0.10 = 2x + 0.15$

b) $2(x + 0.50) = 3(x + 0.25)$

Reflect and Check

3. a) How can you model solutions to equations with variables on both sides using manipulatives or diagrams?

b) Explain other ways that you could model the solutions.

4. Laura and her greyhound, Dash, went home along a trail. Laura strolled at 2 km/h along the shortest route. Dash trotted along at 3 km/h but covered 0.50 km of extra distance by zigzagging. Suggest methods for determining the length of time they took to reach home together. Share your ideas with your classmates.

Link the Ideas

Example 1: Apply Equations of the Form $ax = b + cx$

In a jar of coins, there are 30 fewer quarters than dimes. The value of the dimes equals the value of the quarters. How many dimes are in the jar?

Solution

Let d represent the number of dimes.
The number of quarters is $d - 30$.
The value of the dimes is $0.10d$ dollars.
The value of the quarters is $0.25(d - 30)$ dollars.
The value of the dimes equals the value of the quarters.
An equation that represents the situation is $0.10d = 0.25(d - 30)$.

$$\begin{aligned} 0.10d &= 0.25(d - 30) \\ 0.10d &= 0.25d - 7.5 \\ 0.10d - 0.25d &= 0.25d - 7.5 - 0.25d \\ -0.15d &= -7.5 \\ \frac{-0.15d}{-0.15} &= \frac{-7.5}{-0.15} \\ d &= 50 \end{aligned}$$

There are 50 dimes in the jar.

$$\frac{-7.5}{-0.15} \approx \frac{-8}{-0.2} \approx 40$$

ME

You could use fractions of dollars and write the equation as $\frac{1}{10}d = \frac{1}{4}(d - 30)$.

You could also use cents and write the equation as $10d = 25(d - 30)$

Check:
There are 30 fewer quarters than dimes.
$50 - 30 = 20$
There are 20 quarters in the jar.
Value of dimes: $50 \times \$0.10 = \$5.00$
Value of quarters: $20 \times \$0.25 = \$5.00$
The dimes and quarters have equal values, as stated in the problem.

Show You Know

In a jar of coins, there are 20 more nickels than quarters. The value of the nickels equals the value of the quarters. How many quarters are in the jar?

Example 2: Apply Equations of the Form $ax + b = cx + d$

Alain has \$35.50 and is saving \$4.25/week. Eva has \$24.25 and is saving \$5.50/week. In how many weeks from now will they have the same amount of money?

Solution

Let the number of weeks from now be w.
In w weeks, Alain will have $35.50 + 4.25w$ dollars.
In w weeks, Eva will have $24.25 + 5.50w$ dollars.
In w weeks, Alain and Eva will have the same amount of money.
An equation that describes the situation is $35.50 + 4.25w = 24.25 + 5.50w$.

Isolate the variable.

$$\begin{aligned}
35.50 + 4.25w &= 24.25 + 5.50w \\
35.50 + 4.25w - 4.25w &= 24.25 + 5.50w - 4.25w \\
35.50 &= 24.25 + 1.25w \\
35.50 - 24.25 &= 24.25 + 1.25w - 24.25 \\
11.25 &= 1.25w \\
\frac{11.25}{1.25} &= \frac{1.25w}{1.25} \\
9 &= w
\end{aligned}$$

Alain and Eva will have the same amount of money in nine weeks.

Check:
In nine weeks, Alain will have \$35.50 + 9 × \$4.25, or \$73.75.
In nine weeks, Eva will have \$24.25 + 9 × \$5.50, or \$73.75.
So, they will have the same amount of money in nine weeks from now.
The solution is correct.

Try a different first step than subtracting $4.25w$ from both sides. Complete your solution and check that you can get the correct final answer.

Show You Know

One Internet café charges \$1 for 15 min and \$0.20 per page for printing. A second Internet café charges \$2 per hour and \$0.25 per page for printing. Suppose you want to use the Internet for one hour. How many pages would you need to print in order to make the two cafés equal in price?

Example 3: Solve Equations of the Form $a(bx + c) = d(ex + f)$

Solve $\frac{1}{3}(2x - 1) = \frac{1}{2}(3x + 1)$ and check.

Solution

$$
\begin{aligned}
6 \times \frac{1}{3}(2x - 1) &= 6 \times \frac{1}{2}(3x + 1) \\
2(2x - 1) &= 3(3x + 1) \\
4x - 2 &= 9x + 3 \\
4x - 2 - 4x &= 9x + 3 - 4x \\
-2 &= 5x + 3 \\
-2 - 3 &= 5x + 3 - 3 \\
-5 &= 5x \\
\frac{-5}{5} &= \frac{5x}{5} \\
-1 &= x
\end{aligned}
$$

Check:

$$
\begin{aligned}
\text{Left Side} &= \frac{1}{3}(2x - 1) \\
&= \frac{1}{3}[2(-1) - 1] \\
&= \frac{1}{3}(-2 - 1) \\
&= \frac{1}{3}(-3) \\
&= -1
\end{aligned}
\qquad
\begin{aligned}
\text{Right Side} &= \frac{1}{2}(3x + 1) \\
&= \frac{1}{2}[3(-1) + 1] \\
&= \frac{1}{2}(-3 + 1) \\
&= \frac{1}{2}(-2) \\
&= -1
\end{aligned}
$$

Left Side = Right Side

The solution, $x = -1$, is correct.

Show You Know

Solve $\frac{3f + 1}{4} = \frac{3 + 2f}{2}$ and check.

Key Ideas

- You can solve and check equations with variables on both sides by applying the algebraic techniques learned in earlier sections.

$$
\begin{aligned}
3(0.5t + 1.3) &= 2(0.4t - 0.85) \\
1.5t + 3.9 &= 0.8t - 1.7 \\
1.5t + 3.9 - 0.8t &= 0.8t - 1.7 - 0.8t \\
0.7t + 3.9 &= -1.7 \\
0.7t + 3.9 - 3.9 &= -1.7 - 3.9 \\
0.7t &= -5.6 \\
\frac{0.7t}{0.7} &= \frac{-5.6}{0.7} \\
t &= -8
\end{aligned}
$$

Check:

$$
\begin{aligned}
\text{Left Side} &= 3(0.5t + 1.3) \\
&= 3[0.5(-8) + 1.3] \\
&= 3(-4 + 1.3) \\
&= 3(-2.7) \\
&= -8.1
\end{aligned}
\qquad
\begin{aligned}
\text{Right Side} &= 2(0.4t - 0.85) \\
&= 2[0.4(-8) - 0.85] \\
&= 2(-3.2 - 0.85) \\
&= 2(-4.05) \\
&= -8.1
\end{aligned}
$$

Left Side = Right Side

The solution, $t = -8$, is correct.

Check Your Understanding

Communicate the Ideas

1. Ken solved the equation $\frac{r}{2} = 3(r + 0.5)$ as shown. Is Ken's solution correct? If not, identify any errors and determine the correct solution.

$$\frac{r}{2} = 3(r + 0.5)$$
$$2 \times \frac{r}{2} = 2 \times 3(r + 0.5)$$
$$r = 6(2r + 1)$$
$$r = 12r + 6$$
$$-11r = 6$$
$$r = -\frac{6}{11} \text{ or } -0.545\ 4\ldots$$

2. Helga and Dora were solving the following problem together.

A boat left Kyuquot and headed west at 15.5 km/h. A second boat left Kyuquot half an hour later and headed west at 18 km/h. For how many hours did the first boat travel before the second boat overtook it?

Both girls used t to represent the time taken by the first boat. However, they disagreed on the equation to model the situation.

Helga's Equation	*Dora's Equation*
$15.5t = 18(t + 0.5)$	$15.5t = 18(t - 0.5)$

a) Explain what the expression on each side of each equation represents.

b) Which equation is correct? Explain.

c) What is the solution to the problem?

Did You Know?

Kyuquot is a village on the west coast of Vancouver Island. The village is the home of the northernmost of the Nuu-chah-nulth First Nations bands. The Nuu-chah-nulth people were formerly known as the Nootka.

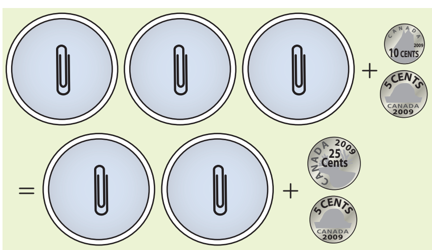
Digital rights not available.

3. a) Pierre decided to try solving $0.5(2x + 3) = 0.2(4x - 1)$ by guess and check. Do you think that this is a good method for solving the equation? Explain.

b) What method would you use to solve the equation?

Practise

4. Write an equation that is modelled by the diagram. Then, solve it.



5. Model the equation $3(x + 0.15) = 2(x + 0.50)$. Then solve it.

For help with #6 and #7, refer to Example 1 on page 323.

6. Solve and check.

a) $0.5x = 1.6 + 0.25x$

b) $\frac{1}{3}y - \frac{1}{2} = \frac{1}{6}y$

c) $7.52 + 3.2a = -6.2a$

d) $-g = 2\frac{1}{2}g - 3$

7. Solve.

a) $\frac{1}{2}n = \frac{2}{5} + \frac{1}{5}n$

b) $-0.2w - 1.1 = 0.3w$

c) $5.1 - 3.5p = -2.3p$

d) $\frac{1}{2}(1 - e) = 1\frac{1}{6}e$

e) $\frac{3}{4}(d + 2) = \frac{2}{3}d$

For help with #8 and #9, refer to Example 2 on page 324.

8. Solve and check.

a) $2.6 + 2.1k = 1.5 + 4.3k$

b) $\frac{1}{6}p - 5 = \frac{1}{2}p + 2$

c) $4.9 - 6.1u = -3.2u - 3.8$

d) $4 + \frac{3}{5}h = -1\frac{2}{5}h - 1$

9. Solve.

a) $0.25r - 0.32 = 0.45r + 0.19$

b) $15.3c + 4.3 = 16.9 - 16.2c$

c) $-\frac{7}{8}k + 2 = 1 - \frac{3}{4}k$

d) $1\frac{1}{2}p + \frac{1}{4} = 2\frac{1}{4}p - \frac{5}{2}$

For help with #10 to #12, refer to Example 3 on page 325.

10. Solve and check.

a) $2(q - 0.1) = 3(0.3 - q)$

b) $\frac{1}{2}(x + 1) = \frac{1}{3}(x - 1)$

c) $0.2(4y + 3) = 0.6(4y - 1)$

d) $\frac{2x - 1}{2} = \frac{2x + 1}{3}$

11. Solve.

a) $4(s + 1.6) = -3(s - 1.2)$

b) $6.2(2g - 3) = 4.2(2g + 3)$

c) $\frac{3}{4}(x + 2) = \frac{2}{3}(x + 3)$

d) $\frac{6m - 3}{5} = \frac{4m - 1}{3}$

12. Solve. Express each answer to the nearest hundredth.

a) $1.2c - 7.4 = 3.4c$

b) $0.59n = 3.2(4 - n)$

c) $4.38 - 0.15x = 1.15x + 2.57$

d) $-0.11(3a + 5) = 0.37(2a - 1)$

Apply

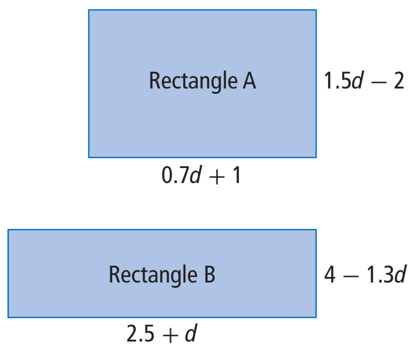
13. A jar contains 76 more pennies than nickels. The total value of the pennies equals the total value of the nickels.

a) How many nickels are there?

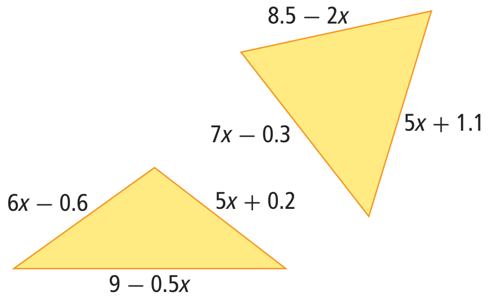
b) What is the total value of all the coins in the jar?

14. Atu now has \$28.50 and is saving \$8.75/week. Beth now has \$104.75 and is spending \$6.50/week from her savings. In how many weeks from now will they have the same amount of money?

15. The two rectangles have equal perimeters. What are the dimensions of each rectangle?



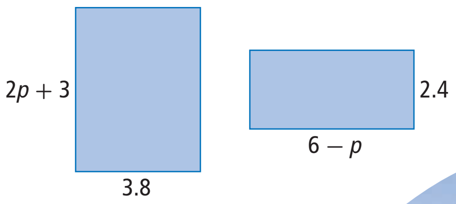
16. a) Determine the value of x so that the two triangles have equal perimeters.



b) Check your solution by evaluating the perimeter of each triangle.

17. Sarah and Rachel are sisters. They leave a park at the same time on their bicycles and ride home along the same bicycle path. Sarah is in a hurry, so she cycles at 15 km/h. Rachel has time to spare, so she cycles at 11 km/h. Sarah gets home 12 min before Rachel. How long did Sarah take to ride home from the park?

18. The two rectangles have equal areas. Determine the area of each rectangle.



19. Elda walked from her home to her friend Niabi's house at 4.5 km/h. When Elda returned home along the same route, she strolled at 3.5 km/h. Elda took a total of 40 min to walk to Niabi's house and back again.

a) How many minutes did Elda take to walk from her home to Niabi's house?

b) How far is it from Elda's home to Niabi's house?

20. Alan's height is $\frac{4}{5}$ of his father's height. Alan's older brother, Ben, is 6 cm taller than Alan. Ben's height is $\frac{5}{6}$ of their father's height. How tall is their father?

21. Members of a cinema club pay $10 to see a movie instead of paying the regular price of $12.50. Annual membership in the club costs $30. What is the least number of movies you would need to see in a year in order to save money by buying a membership?

22. In still water, Jana's motorboat cruises at 16.5 km/h. On the river, the boat travels faster downstream than upstream, because of the current. The boat takes 5 h for a trip upstream, but only 2 h to cover the same distance on the return trip downstream. Determine the speed of the current.

23. Is each a true statement? Explain your reasoning.

a) $1.2(0.5x - 1.8) = 0.8(0.75x - 2.7)$

b) $-0.7(0.45y + 0.6) = -0.5(0.63y + 0.84)$

24. Describe a situation that could be modelled by the equation $3.5x + 1.2 = 4x + 0.9$.

25. Write a problem that can be modelled by an equation with the same variable on both sides. Have a classmate solve your problem.

Extend

26. Solve.

a) $0.25x + 0.75x = 0.8x + 3.5$

b) $\frac{y}{2} - \frac{y}{3} = \frac{y}{5} + 1$

c) $0.5(4d + 3) - 2.6 = 1.5d$

d) $2.6(j - 1) + 0.7 = 1.2(3 - j) + 0.2$

27. Solve.

a) $15.3 - 8.9 - 1.3a = 4.3a + 0.1$

b) $3 - \frac{1}{2}(4 - s) = 2 + \frac{1}{4}(5 + s)$

c) $\frac{1}{3} + \frac{2}{3}(q - 2) = \frac{4}{3}(q + 2) - \frac{5}{3}$

d) $1.5z - 2.1(2z + 3) = 4.2z + 0.3(z + 9)$

28. Solve $2(x + 5) = 3(x + k)$ for k and check the solution.

29. If $m = -0.8$ is a solution to the equation $2(1.8n + m) = m(5 - 2n)$, what is the value of n?

30. Solve.

a) $\frac{2x + 1}{2} + \frac{4x - 5}{3} = -1$

b) $\frac{3y + 4}{5} - \frac{y + 2}{2} = \frac{4y - 1}{10}$

Math Link

The mass of riboflavin in one small serving (75 g) of raw almonds is 0.87 mg less than the mass of riboflavin in $2\frac{1}{2}$ small servings of raw almonds. What is the mass of riboflavin in one small serving of raw almonds?

a) Write an equation that models the situation.

b) Solve the equation using guess and check.

c) Solve the equation by isolating the variable.

d) Which of these solution methods do you prefer? Explain why.

Did You Know?

Riboflavin is another name for vitamin B_2. It helps our digestion and our immune system. A lack of riboflavin can cause skin and eye irritation. Sources of riboflavin include milk, yeast, and eggs.

Chapter 8 Review

Key Words

For #1, fill in each blank by selecting from the terms provided. Terms can be used more than once.

1. In the _____ $\frac{2}{3}k + \frac{1}{2} = -\frac{5}{6}$,

- k is a _____,
- $\frac{2}{3}$ is a _____,
- $\frac{1}{2}$ is a _____, and
- $-\frac{5}{6}$ is a _____.

A numerical coefficient
B variable
C constant
D equation

For #2 and #3, unscramble the letters to complete the statements using key words. Explain the meanings of the key words.

2. Subtraction is the _____ _____ to addition.
N O I S I E T O P O P A T E R O P

3. When solving equations, you can remove brackets using the _____ _____.
O P P R I S I T E T R U B E D Y V I R T

8.1 Solving Equations: $ax = b$, $\frac{x}{a} = b$, $\frac{a}{x} = b$, pages 292–303

4. Model the solution to the equation $\frac{x}{2} = -0.4$ on a number line. What is the solution?

5. Solve and check.

a) $4d = -\frac{2}{5}$

b) $2.68 = \frac{y}{3}$

c) $\frac{3.5}{h} = -0.2$

d) $-7.6u = -14.44$

6. Solve. Express each solution to the nearest tenth.

a) $67.5 = -14.3w$

b) $-\frac{68.3}{k} = -7.6$

7. The density of an object, D, in grams per cubic centimetre, is defined by $D = \frac{m}{V}$. In this formula, m is the mass, in grams, and V is the volume, in cubic centimetres. The density of pure iron is 7.87 g/cm^3.

a) What is the mass of a piece of pure iron with a volume of 5.5 cm^3?

b) What is the volume of a piece of pure iron with a mass of 98.375 g?

8. Raj and Tom are the top scorers on their soccer team. One season, Raj scored one quarter of their team's goals and Tom scored one sixth of their team's goals. Raj and Tom scored a total of 25 goals. How many goals did their team score?

8.2 Solving Equations: $ax + b = c$, $\frac{x}{a} + b = c$, pages 304–313

9. Model the solution to the equation $2x + \frac{1}{12} = \frac{3}{4}$ on a number line. What is the solution?

10. Nona solved the equation $-3.3g + 1.5 = -8.4$ as shown.

$$-3.3g + 1.5 = -8.4$$
$$-3.3g + 1.5 - 1.5 = -8.4 - 1.5$$
$$-3.3g = -9.9$$
$$\frac{-3.3g}{3.3} = \frac{-9.9}{3.3}$$
$$g = -3$$

a) What mistake did Nona make?

b) What is the correct solution?

11. Solve and check.

a) $\frac{t}{1.6} + 5.9 = -3.2$

b) $2.05 = 0.9x - 6.5$

c) $\frac{2}{5} = \frac{2}{3} - \frac{r}{5}$

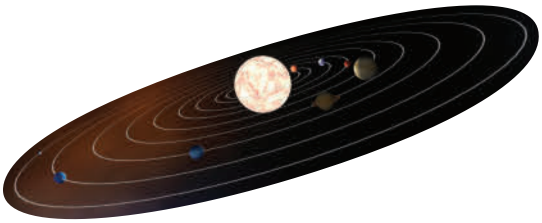
d) $2\frac{1}{2} + 1\frac{1}{6}v = -\frac{5}{6}$

12. Oksana paid a \$14.00 handling charge to buy four Leela Gilday concert tickets on the Internet. The total cost of her order, including the handling charge, was \$153.80. What was the cost of each ticket?

Did You Know?

Leela Gilday was born and raised in Yellowknife, Northwest Territories. She is a singer/songwriter from the Dene Nation. Leela started singing at an early age, and her performing career began at the Folk on the Rocks Music Festival when she was eight years old. Leela now performs across Canada and abroad with her band.

13. The sun takes 25.38 Earth days to rotate once about its axis. This time is 3.92 Earth days less than half the time that Mercury takes to rotate once about its axis. How many Earth days does Mercury take to rotate once about its axis?



8.3 Solving Equations: $a(x + b) = c$, pages 314–321

14. Solve and check.

a) $3(5.8 + e) = -2.7$

b) $-\frac{5}{6} = \frac{r - 4}{3}$

c) $10.1 = \frac{0.9 + h}{-2}$

d) $-\frac{4}{5}(q + 1) = -1\frac{1}{2}$

15. Solve $0.11 = -3(1.93 + m)$. Express the solution to the nearest hundredth.

16. An equilateral triangle has a perimeter of 17.85 m. The side length of the triangle, in metres, is $2k - 0.5$. What is the value of k?

17. Lorna took three of her friends to visit the zoo. The bus fares cost \$5.50 per person. The cost of admission to the zoo was the same for each person in the group. Lorna spent \$109 altogether on fares and admission. What was the cost of each admission?

8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$, pages 322–329

18. Solve and check.

a) $\frac{1}{5}n + \frac{3}{2} = \frac{3}{10}n$

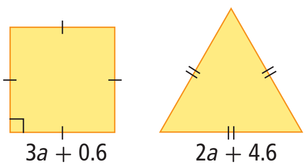
b) $-0.25f = 0.35 - 0.45f$

c) $12.4g + 34.3 = 9.5 - 3.1g$

d) $1.4(2h + 3) = 0.9(3h - 2)$

e) $\frac{3v + 2}{3} = \frac{2v - 1}{4}$

19. The square and the equilateral triangle have equal perimeters.



a) What is the value of a?

b) What is the perimeter of each shape?

20. The perimeter of a rectangle is 3.5 times the length. The width is 2.5 cm less than the length. What is the width?

Chapter 8 Practice Test

For #1 to #4, select the correct answer.

1. What is the solution to the equation $\frac{1}{3} - \frac{3}{2}x = -\frac{1}{6}$?

A $-\frac{1}{9}$ **B** $\frac{1}{9}$

C $\frac{3}{1}$ **D** $\frac{1}{3}$

2. What is the solution to the equation $\frac{-5.2}{t} = -3.25$?

A 1.6 **B** −1.6

C 0.625 **D** −0.625

3. What is the solution to the equation $0.45 - 0.3g = 0.85 + 0.2g$?

A 0.8 **B** −0.8

C 1.25 **D** −1.25

4. Which equation does not have the solution $y = -2$?

A $\frac{y}{4} + 1 = \frac{1}{2}$

B $\frac{7}{8} - \frac{1}{y} = 1\frac{3}{8}$

C $\frac{2y - 1}{4} = \frac{5y - 4}{8}$

D $\frac{2}{3}y + \frac{3}{2} = -\frac{1}{12}y$

Complete the statements in #5 and #6.

5. To solve a linear equation, you isolate the ______.

6. For $2.43 = -0.38v$, the solution expressed to the nearest hundredth is $v =$ ______.

Short Answer

7. Model the solution to the equation $\frac{x}{2} = -\frac{3}{4}$ on a number line. What is the solution?

8. a) Describe the steps you would use to solve the equation $1.5(x + 3) = 0.5(x - 1)$.

b) How would the steps in part a) be different from those you would use to solve $1.5x + 3 = 0.5x - 1$?

9. Solve and check.

a) $\frac{a + 1}{2} = \frac{2a - 1}{5}$

b) $2.8(3d - 2) = -12.32$

10. Solve. Express each solution to the nearest tenth.

a) $-13.9x = 5.7 - 12.5x$

b) $0.8(2s + 3) = -0.6(5s - 2)$

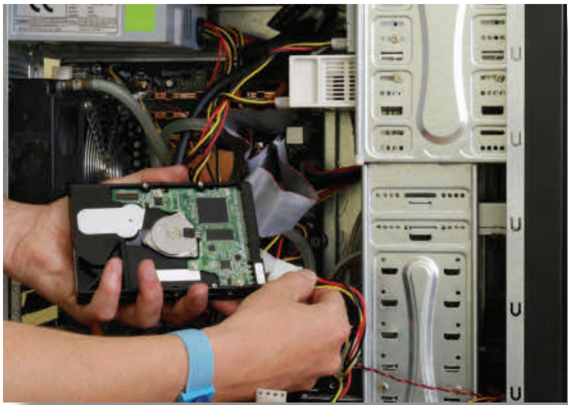
11. Precipitation is moisture that falls in the form of rain or snow. The relationship between the depth of rain, r, and the depth of snow, s, that results from equal quantities of precipitation is $\frac{r}{s} = 0.1$.

a) If a storm delivers 15.5 cm of snow, what depth of rain would result from the same amount of precipitation on a warmer day?

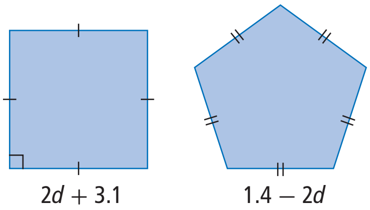
b) If a storm delivers 2.7 cm of rain, what depth of snow would result from the same amount of precipitation on a colder day?

12. Nav is working part time. She pays a monthly fee of \$5.95 for her bank account, plus \$0.75 for each deposit or withdrawal. One month, the total cost of her account was \$12.70. How many deposits or withdrawals did she make that month?

13. Two computer technicians both charge a fee for a home visit, plus an hourly rate for their work. Dana charges a \$64.95 fee, plus \$45/h. Tom charges a \$79.95 fee, plus \$40/h. For what length of service call do Dana and Tom charge the same amount?



14. The square and the regular pentagon have equal perimeters. What is the perimeter of each shape?



Extended Response

15. a) Identify the error in the following solution.

$$\begin{aligned} -3.1(2n + 3) &= 12.3 \\ -6.2n + 3 &= 12.3 \\ -6.2n + 3 - 3 &= 12.3 - 3 \\ -6.2n &= 9.3 \\ \frac{-6.2n}{-6.2} &= \frac{9.3}{-6.2} \\ n &= -1.5 \end{aligned}$$

b) Correct the error in part a) to determine the correct solution. Express your answer to the nearest tenth.

Math Link: Wrap It Up!

The table shows the energy content, in megajoules, of single servings of some common foods.

Food	Serving Size	Energy (MJ)
Brazil nuts (raw)	125 mL	2.03
Buttermilk	250 mL	0.44
Canola oil	15 mL	0.52
Cheddar cheese	45 g	0.76
Corn (boiled)	1 ear	0.35
Lentils (cooked, drained)	250 mL	1.02
Mango (peeled)	1 mango	0.48
Potato (baked)	1 potato	0.94
Salmon (canned)	95 g	0.81

Use data from the table to write a word problem that can be solved using each of the following types of linear equations. Show the complete solutions to your own problems.

Have a classmate check your solutions. Modify your problems or correct your solutions, if necessary.

a) an equation of the form $ax = b$

b) an equation of the form $\frac{x}{a} = b$

c) an equation of the form $ax + b = c$

d) an equation that includes a grouping symbol

e) an equation with the same variable on both sides

Challenges

School Store

A school store is a convenience to both students and teachers. But a school store is also a great opportunity for you. If you volunteer to work in the school store, it gives you real job training.

You be the store worker. How can you use your knowledge of linear equations to help you make business decisions?

1. The wholesale price of a case of 36 fruit bars is $20. If the bars are sold in the store for $0.75/bar, write and solve an equation to determine the amount of profit for the sale of one case of fruit bars.

2. The student leadership team has volunteered to work and promote sales in the store for one week. The profit made during that week from the sale of granola and fruit bars, chocolate milk, and frozen milk treats will be donated to charity. The price and average sales of these items are provided in the table.

Product	Price/Case	Number in Each Case	Average Weekly Sales
Granola bars	$26.40	48	115
Fruit bars	$20.00	36	160
Chocolate milk	$43.20	90	156
Frozen milk treats	$17.98	18	88

a) Determine a price per item so that the store makes a profit of at least $150 during the week. Show your thinking for determining the price for each item.

b) Plan an advertisement that includes price per item.

Did You Know?

In business, many managers use a spreadsheet to solve this type of problem.

Literacy Link

Wholesale price is the price charged for products when you buy large numbers of them from someone who sells to the owners of stores.

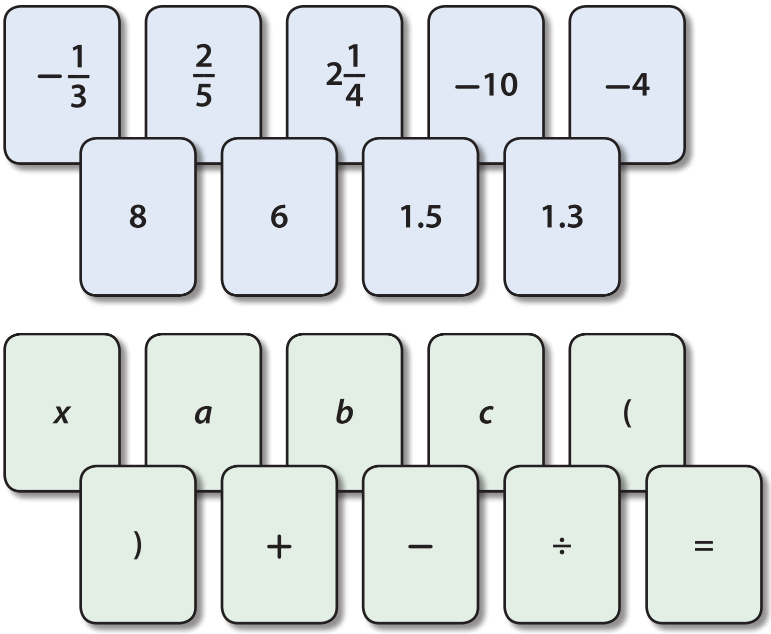
Profit refers to the difference between selling price of goods and what it costs to buy and sell them.

Pair Up, Create, and Solve

Materials
- scissors
- blank paper

In this game, you will solve various equations that you and your partner will make together.

1. Cut a piece of paper into 19 equal-sized cards. You may find it easiest to cut the paper into 20 pieces and discard one. Label the cards as shown.



Turn the cards in the first two rows face down on the table in a random fashion so that the numbers are hidden. Keep the cards in the second two rows face-up.

2. Using the cards in the second two rows, one partner will lay out an equation on the table in one of the following forms:

$ax = b \quad \frac{x}{a} = b \quad ax + b = c \quad \frac{x}{a} + b = c \quad a(x + b) = c$

3. The second partner randomly selects from the overturned cards to replace a, b, and/or c with numerical values, and then solves the resulting equation. Work together to check the answer.

4. Replace the cards in their rows, and switch roles. Repeat this process until you have tried all five equation forms.